Perspectives on Borges' Library of Babel

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Abstract

This study builds on our Bridges 2012 work on *Harmonic Perspective*. Jeannie's artwork explores 3D spaces with an intersecting plane construction. We used the subject of the remarkable "Universe (which others call the Library)" of Jorge Luis Borges. Our study explores the use of harmonic sequences and nets of rationality. We expand on our prior use of harmonics in Apollonian circles to a pencil of nonintersecting circles. Using matrix distortion and ambiguous direction in the multiple dimensions of a spiral staircase in the Library we break from harmonic measurements. We wonder about the contrasting perspectives of unimaginable finiteness and infiniteness.

Figure 1: *Borges' Library of Babel (View of Apollonian Ventilation Shafts).*

Figure 2: *Borges' Library of Babel (Other View).*

Introduction

In 1941, Jorge Luis Borges (1899–1986) published a short story *The Library of Babel* [2] that has intrigued artists, mathematicians, and philosophers ever since. The story plays with ideas of infinity, the very large but finite, the components of immensity, and periodicity. The Library consists of a vast number of hexagonal rooms each with the same number of books. Although each book is unique, each has the same number of pages, lines, and characters. The librarians wander their whole lives through the labyrinth of rooms wondering about the nature of their Universe. Many aspects of the Library are left open to speculation.

Giuseppe Mazzotta defines perspectivism as “a way of assembling various points of view” [7]. This approach accommodates many ways of knowing, perceiving and doing. Shouldn’t the concept of perspective be liberated from the limiting form developed by Renaissance artists and abstracted into the science of projective geometry by mathematicians? After reading William Goldbloom Bloch’s book [1] about key mathematical implications in Borges’ story, we set out to visualize the Library without the traditional window and Renaissance perspective. The viewer moves around the space and even into it. Mirror images are set up to make us question our own position vis-a-vis the Library.

In our 2012 Bridges paper [6] we described the geometry of harmonic perspective, three harmonic constructions, and three artworks illustrating them. In this study we used two additional harmonic constructions and a new artwork also using intersecting planes to advance our continuing search for unorthodox ways to create space perspective. Our process was one of research, discussions, computer images by CJ and construction of the artwork by Jeannie. Some of the questions that challenged us in this project were: Can geometric invariants be used to highlight movement in a geometric space? Can seeing the harmonic relationships in an artwork reveal new aspects of imaginative scope? Let us begin by explaining the mathematics behind the two main techniques of harmonic perspective used in this study.

Harmonic Sequences and Nets of Rationality

On a line, a harmonic sequence [5, p. 32] can be constructed as in Figure 3 by starting with three arbitrary points which we suggestively label 0, 1, and ∞. Draw a line PQ meeting the line 01 at ∞. Draw 0P and 1Q intersecting at point A. Draw line A∞. Draw line 1P intersecting A∞ at B. Intersecting BQ with line 01 gives point 2. By repetition, additional points of the sequence may be constructed.

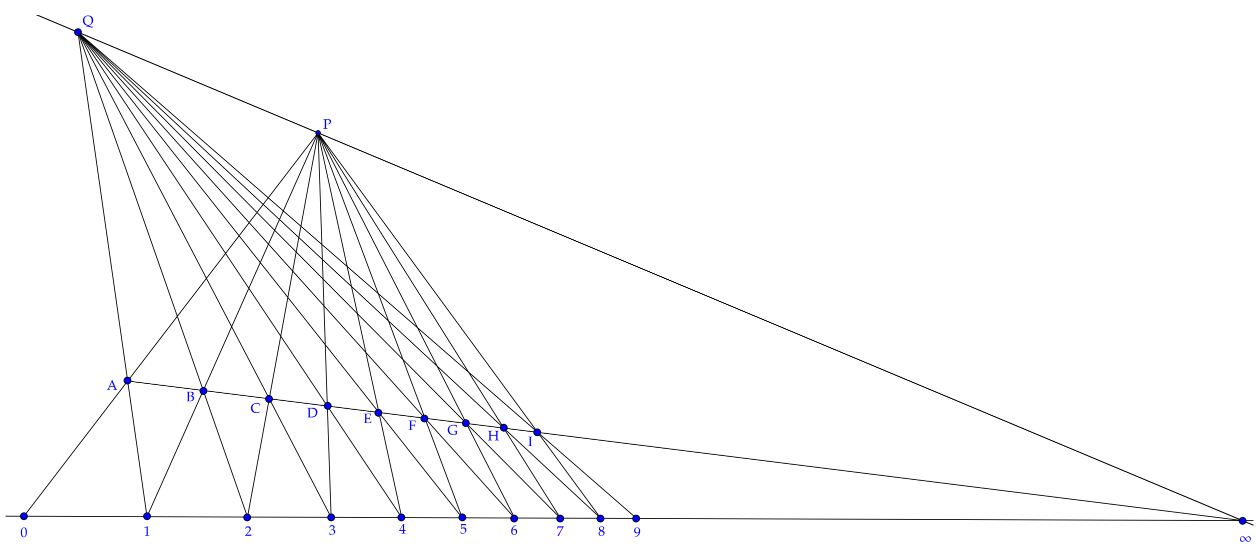

Figure 3 : *Construction of a harmonic sequence.*

A complete quadrangle is formed by six lines joining four points such as ABPQ. The harmonic set H(1∞,02) (read as “2 is the harmonic conjugate of 0 with respect to 1 and ∞”) is given when pairs of “opposite” sides of ABPQ cut a line at two of the intersections of its opposite sides: QA·BP=1, QP·AB=∞; AP·01=0, BQ·01=2 [3, p. 18]. By repeating this with BCPQ, we have H(2∞,13), etc. In this way we can see the fundamental role of harmonics in a projective model of the integers. This geometrical approach allows us to count all $25^{1,312,000}$ books in Borges' Library and thereby measure astronomical spaces in a picture.

It is possible to add, multiply, and divide points geometrically [10, pp. 141–150][3, pp. 172–6][9, pp. 89–91]. Since these constructions do not require continuity, the arithmetic they naturally form is the familiar field of characteristic zero known as the *rational numbers*. By producing another harmonic sequence (which may be metrically incommensurable with the first) on another distinct line and aligning it with a shared point and including its rational points, a planar net of rationality may be developed. Adding another aligned line in another plane will produce a net in three space. Harmonics play a basic part in defining the geometry of both arithmetic and its allied "net of rationality" [10, pp. 84–92][3, pp. 164–8][5, pp. 30–32].

In constructing *Borges' Library of Babel* see Figures 1 and 2, Jeannie used nets of rationality to make a grid to "measure" the space. Scenes in the piece such as the librarian stamping new books were put through multiple harmonic transformations. What could happen if lines meeting at infinity are segmented in a non-harmonic sequence? What if incidence properties at infinity were anomalous? Could the space packing of the hexagonal rooms of the Library be something other than a honeycomb given the unspecified shapes of the spiral staircase and anterooms? Are the floors necessarily on a flat plane? Can measure be inconsistent and direction ambiguous?

The Non-Intersecting Pencil of Apollonian Circles

Apollonius of Perga (c.262 BCE–c.190 BCE) defined a circle as the locus of points whose distance from two fixed points A and B is given by a fixed ratio. As can be seen in Figure 4, the internal and external angle bisectors (the dashed lines from P) of any triangle PAB where P is any point on such a circle cut the line AB internally and externally in the same ratio, that is, harmonically [4, pp. 88–89, 242] [8, pp. 14–15]. By varying the ratio, a pencil of nonintersecting circles is generated. The pencil includes the circle of radius ∞ as a vertical line (not shown). Jeannie used a selection of circles from such a pencil to represent the layout of the rooms and ventilation shafts in the Library (see the non-intersecting circles in Figures 1 and 2).

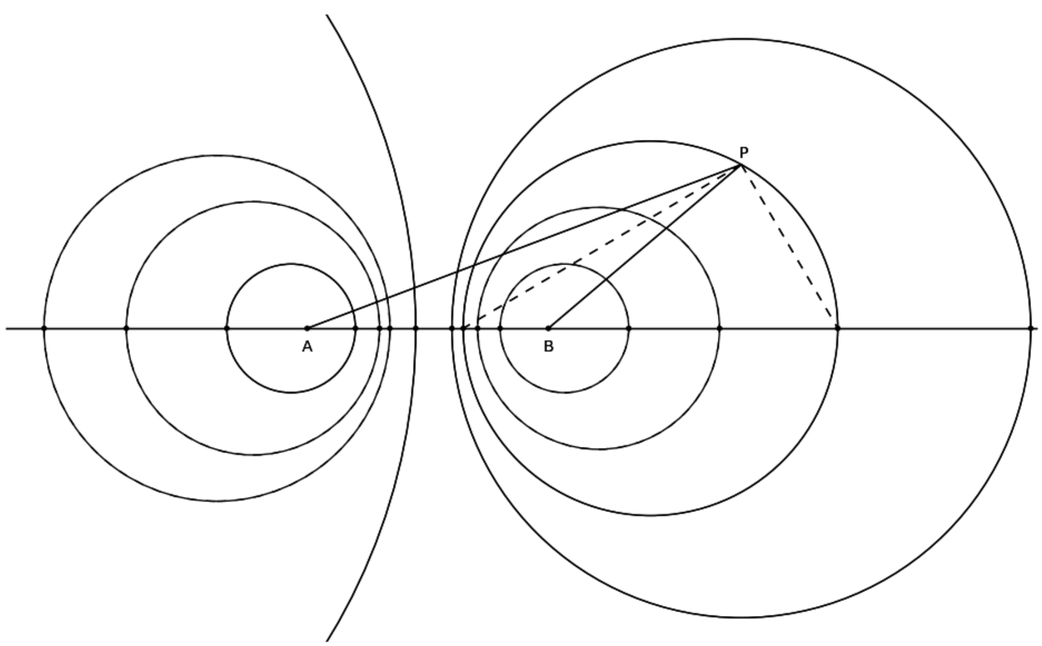

Figure 4: *Pencil of Apollonian Circles.*

Perspectives on *Borges' Library of Babel*: Discussion

An artwork is a tool for the imagination. Our work on this project has helped us exercise our imagination in considering projective geometry and its fundamental notion of harmonics, the role of geometry in providing tools for artistic representation, and the interplay between art and geometry in attempting to imagine and represent imaginary worlds such as Borges' Library. We would like to reflect on perspectives in the unimaginable world of the Library.

In Borges' enigmatic short story, much hinges on the idea that books in the Library contain every possible combination of twenty-five characters resulting in a combinatorial explosion. In Figure 2 we see an open book with the Spanish text "algarabia absoluta" meaning absolute gibberish. This perspective on a "complete" Library is supported by Bloch's analysis [1, pp. 30–44] which concludes that such a Library cannot be catalogued because it is too vast: almost everything in it is nonsense as Borges explains in the story itself. Is meaning simply statistical anomaly? Can art represent both meaning and meaninglessness?

Are there other ways to imagine and represent the starkly unimaginable immensity of the colossal Library? At the end of the story, the narrator suggests the Library is periodic [2, p. 10]. Could a monumental but repetitive form replace the infinite with concrete modularity? Is the nature of periodic systems, finite, infinite, or both/neither? In projective geometry the infinite is made finite by treating ∞ as an actual point on the page as in Figure 3. In a harmonic sequence, the periodic structure of the repeated quadrangles and the harmonic sets they define show the way the natural numbers lead to infinity by a sort of periodicity.

Does the particularity of huge numbers of unique books in hexagon after hexagon suggest a concrete periodicity that may be more "real" than the instantaneous span of an unimaginably vast space by the finite word "infinity"? Is the limit process associated with infinity a gloss or oversimplification of the ontology of Borges' Library? And of our Universe? Can the concrete incidences in harmonic constructions support us in imagining endless periodicities? Which is more foreboding: the mysteries of an infinite Universe or an unimaginably large but finite Library whose scope is also unknowable? Is it possible to assemble such conflicting points of view into an artwork? How can we get perspective on perspective?

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