

# Harmonic Perspective

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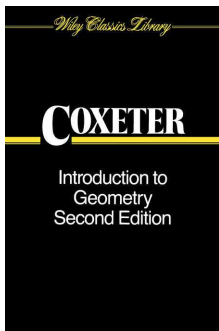
Presentation to ***Bridges Towson 2012: Mathematics, Music, Art, Architecture, Culture*** at Towson University in Towson, MD, USA

Landing Page for Harmonic Perspective:  
<http://www.CJFearnley.com/HarmonicPerspective>

# A Coxeter Inspired Journey

“In **Projective Geometry** ... [there are] no circles, no distances, no angles, no intermediacy [betweenness] and no parallelism.”

— H. S. M. Coxeter, *Introduction to Geometry* (1989), p. 229.



# The Duality Principle

- Projective Geometry may be **the geometry of fundamental duality**
- Duality properties vary by dimension:
  - In 2D, point is dual with line.
  - In 3D, point is dual with plane and line is self-dual.

- 2D Duality Dictionary:

Point (Vertex)	$\longleftrightarrow$	Line (Sides)
“Lies on”	$\longleftrightarrow$	“Pass through”
Join $\vee$	$\longleftrightarrow$	Meet $\wedge$ (intersect)
Concurrent	$\longleftrightarrow$	Collinear

# Projective Geometry

In addition to the references in our paper, my recent essay *“Models of Projective Geometry”* catalogs a bunch of different ways to think of projective geometry:

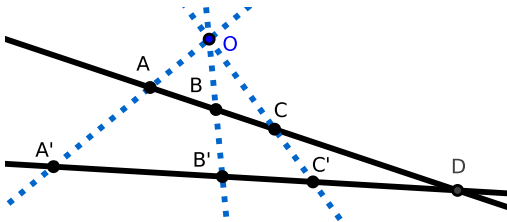
<http://blog.cjfearnley.com/2012/06/02/models-of-projective-geometry/>

# A Reference to Harmonic Perspective

The idea of **Harmonic Perspective** was found in J. L. S. Hatton.  
*The Principles of Projective Geometry Applied to the Straight  
Line and Conic* (1913), pp. 84–5.

# Point Perspective

- Point Perspective: when corresponding points along two lines are paired through a center (pencil).

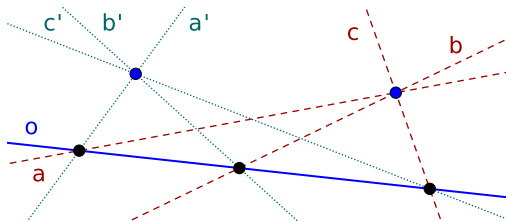


Point Perspective. Two lines are perspective from a point if corresponding points on each line lie along lines through

the *center* of perspective. We write  $ABCD \overset{O}{\wedge} A'B'C'D$ .

# Line Perspective

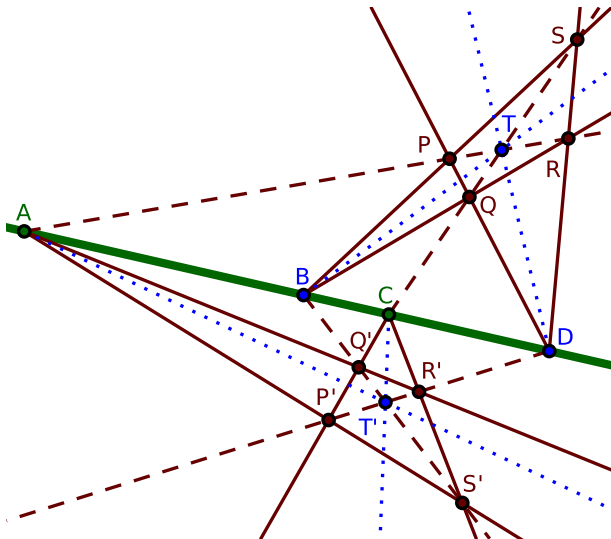
- Line Perspective: when corresponding lines in two pencils are paired through an axis (a line or a range of points).



Line Perspective. Two points are perspective from a line if corresponding lines in each pencil meet at points along the

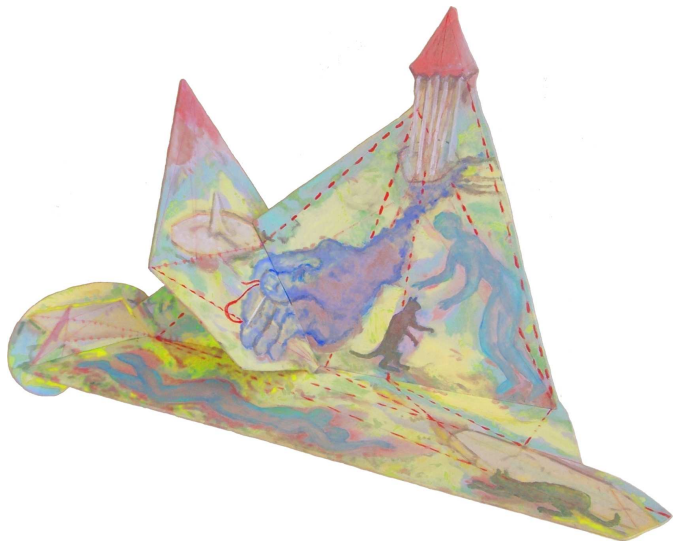
*axis of perspective*. We write  $abc \overset{o}{\wedge} a'b'c'$ .

# Harmonics and Quadrangles

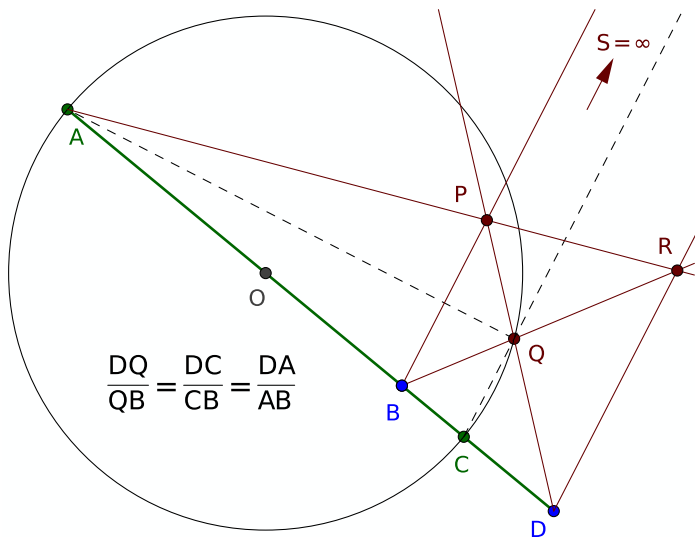




# Here Kitty

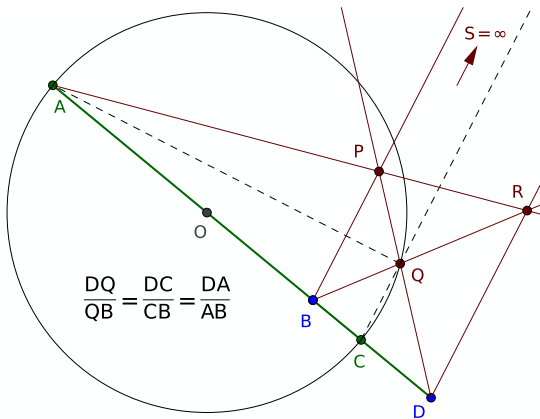


# The Circle of Apollonius



# Harmonic Ratios I

In a harmonic ratio, one outer segment is to the central segment as the whole is to the other outer.



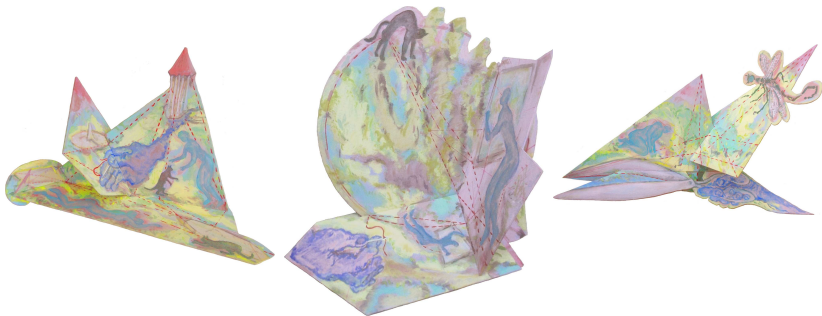
# Harmonic Ratios II

In a harmonic ratio, the outer segment is to the central segment as the whole is to the other outer.

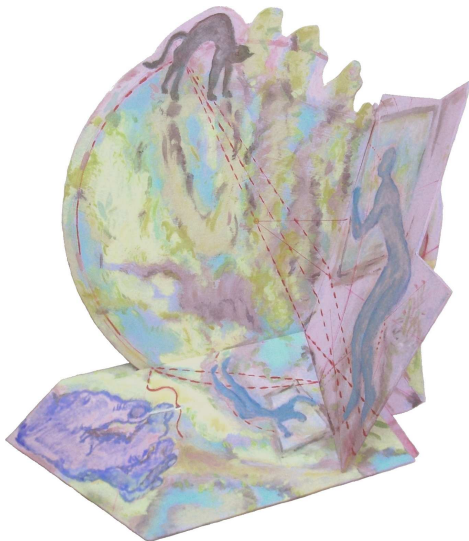
$$\frac{\text{person}}{\text{cat}} = \frac{\text{hand}}{\text{needle}}$$

$$\frac{\text{person}}{\text{window}} = \frac{\text{window}}{\text{cat}}$$

$$\frac{\text{person}}{\text{scissor}} = \frac{\text{insect}}{\text{hand}}$$



# Inside Out





# Intentional Cut



# Unanswered Questions

- Is it possible to design art based on a geometrical invariant such as harmonics?
- Can harmonic perspective help us understand the simultaneous perceptions of large and small, near and far objects?
- What qualities can harmonic perspective interject into a work of art?



# Conclusion

Perspective and harmonics and their interrelationships identify subtleties of spatial experience that speak to the power and importance of projective geometry.

# Thank You

Thank You!

Any Questions?

Landing Page for our Harmonic Perspective work including our  
Bridges 2012 Paper & Presentation:

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