Discussion Questions and Problems for "Variations of the Möbius Band to Explore the Nature of Homeomorphism"

(Revision: November 12, 2017[1])

On 25 November 2017 *Math Counts* will discuss "Variations of the Möbius Band to Explore the Nature of Homeomorphism"[2]. These problems are based on Chapters 3-4 of Stephen Barr's fun book "Experiments in Topology". Your attempts to address these questions and problems will help guide our discussion. If the book or these questions are unclear, please ask for help on the event page[2].

1. According to chapter 1 of Barr's book, what is the rule for homeomorphically distorting one surface into another? What caveats does Barr mention for using his rule? How does Barr's definition compare to Wikipedia's definition of *homeomorphism*?
2. According to chapter 1 of Barr's book, what is the definition for a simply connected surface? How does Barr's definition compare to Wikipedia's definition of a *simply connected space*?
3. What is the property of orientability? Which of the sphere, disk (the 2D surface bounded by a circle), cylinder, torus, and Möbius band are orientable and which are nonorientable? How can you explain orientability with paper models of topological surfaces?
4. In a paper model of a Möbius strip, what happens when you cut through the middle of the strip? What are the resulting surface(s)? How many sides, edges, and separate pieces are there? What logic explains this behavior?
5. Can one build a paper model of a Möbius strip whose width is greater than its length (let us define the width to be the length of each of the two opposite edges of the rectangular strip of paper that are glued together when making the Möbius band)? That is, can the width to length ratio (width/length) ever exceed 1? Note: the length of a Möbius band might be said to be the length of its one and only edge.
6. In the Möbius strip construction using a folded 60° degree angle to form a hexagon shown in Figures 4 and 5 of chapter 3 on p. 42, what is the ratio of width to length?
7. In the Möbius strip construction in Figure 7 on p. 43, what is the width to length ratio?
8. In the Möbius strip construction in Figure 9 on p. 44, what is the width to length ratio?
9. In the Möbius strip construction in Figure 13 on p. 46, what is the width to length ratio?
10. In the Möbius strip construction in Figure 14 on p. 47, what is the width to length ratio?
11. How can one build a paper model of a Möbius band from a square strip of paper?
12. What is the maximum width to length ratio for a paper model of a Möbius strip?
13. How can one build such a "minimum length" Möbius band from paper?

[1]Note: Please let me know of any difficulties. There may be a revised version correcting issues if any are found. Compiled by CJ Fearnley. `http://blog.CJFearnley.com`.

[2]`http://www.meetup.com/MathCounts/events/243186341`

14. What is the width to length ratio in your "minimal length" paper Möbius strip model(s)?
15. What is the width to length ratio of the minimal length paper Möbius strip which can still be cut along the centerline of the strip (see question 4)? Why does the property of cutting a Möbius strip through its centerline fail when the width to length ratio exceeds that amount?
16. In considering the variations on a Möbius band in chapter 3, how should we think about the connectedness of the joined edge of a Möbius band? How is the homeomorphic property preserved in each of these variations?
17. In considering the sequence of experiments in chapter 3 where Möbius strips of decreasing length to width ratios (or increasing width to length ratios) are considered, what did you learn about the nature of homeomorphism and paper representations of topological surfaces?
18. In the text on p. 49 and the associated Figures 15 and 16, a puzzle about cutting a Möbius strip into two equal area pieces is described. What is the solution to the puzzle? What does it mean for the cut to start on the edge? What logic explains the puzzle? What did you learn from this puzzle?
19. Why are the paper models of the Klein Bottle and Projective Plane in Figures 2 and 3 on p. 51 considered troublesome? Is the difficulty related to the subtleties of connectedness in the definition of a homeomorphism? How would you explain the problem?
20. Can a Möbius strip be constructed from an annulus with a radial cut through its bounding circles as in Figure 4 on p. 52?
21. Can a Möbius strip be constructed from a disk with a radial cut to its center?
22. Can a Möbius band be constructed from a semicircle as in Figure 11 on p. 55?
23. Can a Möbius strip be constructed from a 30° sector of a circle?
24. In considering the variations on a Möbius band in chapter 4, how should we think about the distortion and connectedness in the joined edge of the strip of paper used to make a Möbius band? How is the homeomorphic property preserved in each of these variations?
25. In considering the sequence of experiments discussed in chapter 4 where Möbius strips with various extents of connectedness are considered, what did you learn about the nature of homeomorphism and paper representations of topological surfaces?
26. How do the properties of simply connected surfaces and orientability play a role in understanding the variations on a Möbius band considered in chapters 3 and 4?
27. Given the considerations in chapters 3 and 4, what subtleties, limitations, and caveats must we heed about the nature of the distortions allowed and the requirements for connectedness and continuity in Stephen Barr's definition of a homeomorphism?