Harmonic Perspective

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Landing Page for Harmonic Perspective:
http://www.CJFearnley.com/HarmonicPerspective
A Coxeter Inspired Journey

“In Projective Geometry ... [there are] no circles, no distances, no angles, no intermediacy [betweenness] and no parallelism.”

Projective Geometry may be the geometry of fundamental duality

Duality properties vary by dimension:
- In 2D, point is dual with line.
- In 3D, point is dual with plane and line is self-dual.

2D Duality Dictionary:
- Point (Vertex) \leftrightarrow Line (Sides)
- "Lies on" \leftrightarrow "Pass through"
- Join \lor \leftrightarrow Meet \land (intersect)
- Concurrent \leftrightarrow Collinear
In addition to the references in our paper, my recent essay “Models of Projective Geometry” catalogs a bunch of different ways to think of projective geometry:

http://blog.cjfearnley.com/2012/06/02/models-of-projective-geometry/
Point Perspective

- Point Perspective: when corresponding points along two lines are paired through a center (pencil).

Point Perspective. Two lines are perspective from a point if corresponding points on each line lie along lines through the center of perspective. We write $ABCD \overset{O}{\sim} A'B'C'D$. 
Line Perspective: when corresponding lines in two pencils are paired through an axis (a line or a range of points).

Line Perspective. Two points are perspective from a line if corresponding lines in each pencil meet at points along the axis of perspective. We write $abc \overset{o}{\wedge} a'b'c'$. 
Harmonics and Quadrangles
Here Kitty
The Circle of Apollonius

\[
\frac{DQ}{QB} = \frac{DC}{CB} = \frac{DA}{AB}
\]
Harmonic Ratios I

In a harmonic ratio, one outer segment is to the central segment as the whole is to the other outer.

\[
\frac{DQ}{QB} = \frac{DC}{CB} = \frac{DA}{AB}
\]
Harmonic Ratios II

In a harmonic ratio, the outer segment is to the central segment as the whole is to the other outer.

\[
\frac{\text{person}}{\text{cat}} = \frac{\text{hand}}{\text{needle}} \quad \frac{\text{person}}{\text{window}} = \frac{\text{window}}{\text{cat}} \quad \frac{\text{person}}{\text{scissor}} = \frac{\text{insect}}{\text{hand}}
\]
Inside Out
Harmonic Perspective

CJ Fearnley/Jeannie Moberly, Math/Art
Intentional Cut
Unanswered Questions

- Is it possible to design art based on a geometrical invariant such as harmonics?
- Can harmonic perspective help us understand the simultaneous perceptions of large and small, near and far objects?
- What qualities can harmonic perspective interject into a work of art?
Perspective and harmonics and their interrelationships identify subtleties of spatial experience that speak to the power and importance of projective geometry.
Thank You

Thank You!

Any Questions?

Landing Page for our Harmonic Perspective work including our Bridges 2012 Paper & Presentation:
http://www.CJFearnley.com/HarmonicPerspective