This work explores artistic representation and perspective with the subject of Jorge Luis Borges' remarkable "Library of Babel".

Next Slide: (Our 2012 Bridges paper: "Harmonic Perspective")

It builds on our 2012 Bridges paper on *Harmonic Perspective* which provided the geometrical toolkit for our exploration.

Next Slide: (Perspectivism)

One of our interests is perspective.

Giuseppe Mazzotta defines perspectivism as "a way of assembling various points of view". Points of view and perspective may affect knowing, doing, and perceiving.

Jeannie:

Shouldn't the concept of perspective be liberated from the limiting form developed by Renaissance artists and later abstracted into the science of projective geometry by mathematicians?

Next Slide: (The Library of Babel)

The subject of this exploration of perspective, perspectivism, and the geometry of harmonics is the remarkable "Universe (which others call the Library)" described in the 1941 short story "The Library of Babel" by Jorge Luis Borges.

Next Slide: (The Unimaginable Mathematics of Borges' Library of Babel)

This book by William Goldbloom Bloch is an accessible, fun survey of the many mathematical surprises lurking in The Library. I highly recommend it.

Next Slide: (View of Apollonian Ventilation Shafts)

Jeannie:

The artwork is mixed media on hardboard using an intersecting plane construction.

Next Slide: (Another View of "Borges' Library of Babel")

Jeannie:

I used some ideas from our harmonics study to imagine what the almost infinite library might look like: a layout that might not be honeycombed, spiral staircases that went every which way, multiple vanishing points, mirror translations, and a librarian (based on CJ) that goes on stamping books that can never be counted.

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Infinite Library: A Poem:

¡Algarabia absoluta!
I'm lost in a library
of hexagonal rooms.
Central air shafts extend
with a book falling
through the room
to the room below
and below.
It falls endlessly.
It never comes to a stop.
. . . I'm lost . . .

Can I find myself in a book?
32 books on a shelf
of a bookcase with 5 shelves
with 4 bookcases per room
quite a manageable 640 books per room.
If I leave this room
PAUSE CJ: Apollonian Circles
to follow a shadow
on the stairs,
I go up,
I go down,
all the same
no direction, no end.

The book will tell me . . .
. . . but which book?
25 to the 1,312,000th power
books in the library
Which page?
Which word?
Can I find myself in a word
or an extreme number?

The Librarian stamps the book
and stamps the next.
He can see the number
of characters in the universe
Stamp,
stamp,
stamp.

He's reading on the toilet.
The words go in and out
following the numbers.
He reads
and reads
absolute gibberish.

Next Slide: (A Harmonic Sequence: Constructing the Integers Geometrically)

CJ:

In our 2012 Bridges paper we described a number of harmonic constructions. There are many others to explore. One that we did not explore last time is the "harmonic sequence" which also provides a way to construct the integers geometrically.

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Jeannie will demonstrate as I try to describe the construction.

Pick three arbitrary collinear points.

We could label them suggestively 0, 1, ∞.

Draw another line PQ meeting the first line at the point labeled ∞.

Draw the line joining point 0 with point P and the line joining point 1 with point Q. Note the point of intersection, A.

Draw a line from that point of intersection to the point ∞.

Draw the line joining the point 1 with the point P which intersects the line A∞ at the point B.

Intersect the line BQ with our starting line 01 to find 2.

In projective geometry, the complete quadrangle ABPQ defines a relationship between the pair 0 and 2 with respect to the pair 1 and ∞. This is one of many harmonic properties.

By repetition, additional points of the sequence may be constructed.

Jeannie: From the artist's point of view, the harmonic part of this perspective drawing is that the segments $\frac{12}{01} : \frac{2\infty}{0\infty}$ and this pattern of proportions is replicated endlessly.

CJ: From a mathematical point of view, we can count all $25^{1,312,000}$ books in Borges' Library on a finite sheet and thereby measure astronomical spaces in a picture.

Does that mean that infinity is concrete and bounded?

Next Slide: (Nets of Rationality)

It is possible to geometrically add, multiply, and divide the points of a harmonic sequence. In this way we can construct all of the rational numbers on a line.

If we produce another harmonic sequence (possibly incommensurable with the first) on another distinct line and align them with a shared point, we get a planar "net of rationality."

Another repetition will give a 3-dimensional "net of rationality".

Jeannie:

I used such nets to "measure" the space while laying out my artwork.

Jeannie: (Erase labels to prepare for Apollonian circles)

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Next Slide: (Constructing a Pencil of Apollonian Circles)

Apollonius of Perga defined a circle as the locus of points whose distance from two fixed points A and B is given by a fixed ratio. We will find a deep connection between these circles and harmonics. Jeannie will demonstrate.

Each of the colorful triangles has a base of constant length and two legs in the same proportion. The blue lines are a second series of triangles with the same base but whose legs are in another fixed proportion.

When a series of triangles with these fixed proportions are laid with their base on the segment AB, the locus of their third vertex (the Ps) forms the circle of Apollonius whose center lies on the line AB. The circle cuts the segment AB internally and externally in the same fixed ratio. This is a basic metrical property of harmonic sets and their connection to the circle of Apollonius.

Jeannie: By putting the triangles on AB upside-down, more points on the circle of Apollonius may be constructed.

By varying the ratio, a pencil of nonintersecting circles is generated. The pencil includes the circle of radius ∞ as a vertical line (not shown).

Next Slide: (Apollonian Circles in "Borges' Library of Babel")

Jeannie:

To represent the layout of the rooms and ventilation shafts in The Library, I used circles from non-intersecting pencils of Apollonian circles.

Next Slide: (Conclusion and Questions)

Our project used harmonics as a tool to explore points of view, perspective, perspectivism, and artistic representation. We applied these to the remarkable Library created by Jorge Luis Borges.

Artworks are tools for the imagination.

Borges' short story explores the enormous but finite and the infinite. At the end of the story, the narrator suggests the Library is periodic. Bloch in his book explores what he calls "The Grand Pattern", an arrangement of all possible random arrangements of the $25^{1,312,000}$ books in a periodic Library.

These are mind-boggling ideas.

The particular and the abstract, the finite and the infinite: these ideas are severely tested by The Library. Is this formidable Borgesian invention a good testing ground for exploring artistic representations of the unknowable, the unimaginable, and the ineffable? Can we assemble the points of view of such abstract ethereal concepts into artifacts (or artworks)?

How can we get perspective on perspective?

Next Slide: (Thank You)

Both: Thank You