# Discussion Questions and Problems for "Topological Surfaces from a Strip of Paper"

(Revision: October 26, 2017[1])

On 28 October 2017 *Math Counts* will discuss Topological Surfaces from a Strip of Paper[2]. These problems are based on Chapters 1-3 of Stephen Barr's fun 238 page book "Experiments in Topology". Your attempts to address these questions and problems will help guide our discussion. If the book or these questions are unclear, please ask for help on the event page[2].

1. According to Barr's book, what is the rule for homeomorphically distorting one surface into another?
2. Compared to a topologically ideal surface, what properties does a sheet of paper have that make it inherently non-topological?
3. What artistry can we employ in the building of models of topological surfaces from strips of paper to rectify these deficiencies, at least to some extent?
4. What topological properties are impossible to represent with a physical sheet of paper as a model of a topological surface?
5. Do you agree with Stephen Barr that it is impossible to represent a topological sphere (considered as a 2D surface) with a sheet of paper? Why? Why not?
6. How can you represent a topological plane, cylinder, torus, and Möbius strip with a strip or sheet of paper?
7. How many joints (taping or glueing or otherwise joining of edges together) are needed in a paper representation of a plane, cylinder, torus, and Möbius strip?
8. If a sheet of paper is said to have 2 sides (faces or surfaces or 2D extents: the front and back, for instance) and 4 edges (the top, bottom, left and right), how many sides and edges are in a paper model of a plane, cylinder, torus, and Möbius strip?
9. What are the numbers of vertices, edges, and faces (sides) in a paper model of a plane, cylinder, torus, and Möbius strip? Compute F-E+V for each of your paper models where F represents the number of faces or sides, E is the number of edges or joints where two faces meet, and V represents the number of vertices or crossings where two or more edges meet.
10. Which of the following surfaces are simply connected: the sphere, a plane, a cylinder, a torus, an annulus, a Möbius strip, a mug with a handle, and a disk (the 2D surface bounded by a circle)? Why?
11. What is the topological property of orientability? Which of the topological sphere, plane, cylinder, torus, and Möbius strip are orientable and which are nonorientable? Do your paper models of these topological surfaces exhibit the orientable property? How?

[1]Note: Please let me know of any difficulties. There may be a revised version correcting issues if any are found. Compiled by CJ Fearnley. `http://blog.CJFearnley.com`.

[2]`http://www.meetup.com/MathCounts/events/243186341`

12. Which of the following topological surfaces are homeomorphic to another surface in the list: a sphere (as a 2D surface), a plane, a cylinder, an annulus, a torus, a Möbius strip, a mug with a handle, and a disk? Why?

13. In a paper model of a Möbius strip, what happens when you cut through the middle of the strip? What are the resulting surface(s)? How many sides, edges, and separate pieces are there? What logic explains this behavior?

14. Can one build a paper model of a Möbius strip whose width is greater than its length (let us define the width to be the two opposite ends of the rectangular strip of paper that are joined along their full extent when making the Möbius band)? That is, can the width to length ratio (width/length) ever exceed 1?

15. In the Möbius strip construction using a folded 60° degree angle to form a hexagon shown in Figure 4 of chapter 3 on p. 42, what is the ratio of width to length?

16. In the Möbius strip construction in Figure 7 on p. 43, what is the width to length ratio?

17. In the Möbius strip construction in Figure 9 on p. 44, what is the width to length ratio?

18. In the Möbius strip construction in Figure 13 on p. 46, what is the width to length ratio?

19. In the Möbius strip construction in Figure 14 on p. 47, what is the width to length ratio?

20. How can one build a paper model of a Möbius strip from a square strip of paper?

21. What is the maximum width to length ratio for a paper model of a Möbius strip?

22. How can one build such a "minimum length" Möbius strip from paper?

23. What is the width to length ratio in your "minimal length" paper Möbius strip model(s)?

24. What is the width to length ratio of the minimal length paper Möbius strip which can still be cut through the middle of the strip? Why does this property of cutting a Möbius strip through the middle fail when the width to length ratio exceeds that amount?

25. In considering the sequence of experiments discussed in chapter 3 where Möbius strips of decreasing length to width ratios (or increasing width to length ratios) are considered, what did you learn about the nature of topology, geometry, and paper representations of topological surfaces?

26. In the text on p. 49 and the associated Figures 15 and 16, a little puzzle about cutting a Möbius strip into two equal area pieces is described. What is the solution to the puzzle? Why does the cut line shown in Figure 16 fail to work as intended? What does that cut actually do? Why must the cut start on the edge? What does it mean for the cut to start on the edge? What logic explains the puzzle? What did you learn from this puzzle?