Discussion Questions and Problems for "Experiments in Topology: Dissecting The Klein Bottle"

(Revision: February 19, 2018[1])

On 24 February 2018 *Math Counts* will discuss "Topological Experiments: The Conical Möbius Band & the Klein Bottle"[2]. The following questions and problems are based on Stephen Barr's fun book "Experiments in Topology". Your attempts to address them will help guide our discussion. If the book or these questions are unclear, please ask for help on the event page[2].

1. Inspired by the book, the questions that follow, or your own initiative, what additional experiments did you attempt? What did you learn from these supplemental experiments?

2. According to chapter 1 of Barr's book, what is the rule for homeomorphically distorting one surface into another? What caveats does Barr mention for using his rule? How does Barr's definition compare to Wikipedia's definition of *homeomorphism*?

3. In the description of the Klein bottle on pp. 34–35, the order of joining the opposite edges is discussed. Why is it that it seems impossible to make the half-twist Möbius join first? What is it about the order of operations that makes doing the untwisted join first easier to imagine?

4. How would you describe a Klein bottle? Where is its hole? Where is its inside and its outside? What is the nature of its self-intersection? Is the self-intersection integral to the idea of a Klein bottle? Why or why not? Does the Klein bottle have a boundary (or an edge)? How would you explain the idea of a Klein bottle to a child?

5. How can one see and understand the nonorientability of the Klein bottle? How can you appreciate its nonorientability in a paper model? How can we understand the nonorientability from the connectedness diagram in figure 18 on p. 34? Does the text on pp. 62–63 clarify the matter? How would you convince a child that it is nonorientable?

6. In Fig. 22 and 23 on p. 37 and the surrounding text, a way to build a symmetrical Klein bottle is described. How can you build it with paper? How should we interpret this model given the statements in the text on p. 38 that "the surface passes through itself" and "That is to say, in intersecting, neither plane interrupts the continuity of the other"?

7. On p. 63, the book asks "What happens when a Klein bottle is cut in two?" What do you think? How many cases need to be considered?

8. How can you interpret the model in Fig. 5 on p. 65 (which is dissection #1 on p. 75) as a Klein bottle? What is joined or glued together and what is cut apart or separated or left open? If we regard the line CC' and the join between AB' with BA' as cuts, how can you interpret the result? How would you describe the dissection and its results? What information about the situation can be gleaned from the top view of the main joints? And from the connectivity diagram?

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[1]Note: Please let me know of any difficulties. There may be a revised version correcting issues if any are found. Compiled by CJ Fearnley. `http://blog.CJFearnley.com`.

[2]`https://www.meetup.com/MathCounts/events/247238814/`

9. How can you interpret the model in Fig. 10 on p. 68 (dissection #2 on p. 76) as a Klein bottle? What is joined or glued together and what is cut apart or separated or left open? If we regard the joint between the edges AB' and $A'B$ as a cut, how can you interpret the result? How would you describe the dissection and its results? What information about the situation can be gleaned from the top view of the main joints? And from the connectivity diagram?

10. How can you interpret the model referred to in dissection #3 on p. 76 as a Klein bottle? What is joined or glued together and what is cut apart or separated or left open? In making the indicated cuts, how can you interpret the result? How would you describe the dissection and its results? What information about the situation can be gleaned from the top view of the main joints? And from the connectivity diagram?

11. Apply all of the questions in item 10 to dissection #4 on p. 76.

12. Apply all of the questions in item 10 to dissection #5 on p. 77.

13. Apply all of the questions in item 10 to dissection #6 on p. 77.

14. In considering the six dissections of the Klein bottle summarized in the list on pp. 75-77, what is the significance of the top views of the joint? And of the connectivity diagrams? What is the result of each dissection? How can we understand how each dissection produces its result? What do the dissections reveal about the topology of the Klein bottle?

15. Which of the six dissections is called the Slipped-disk Klein bottle (see p. 75)?

16. Draw lines representing cuts for each of the six dissections on the steam cabinet model (or any other 3D model) of a Klein bottle (the steam cabinet model is described on p. 72 and in the appendix on pp. 202-203)?

17. In considering the dissections and related experiments with the Klein bottle in Chapter 5, what is the topological significance of the chapter? What did you learn about thinking topologically? What did Barr intend for us to learn?

18. Given Stephen Barr's book so far, all of the experimentation and thinking you have done related to the book and our event(s), what observations, realizations, understandings, and insights have you had about the nature of topology, homeomorphism, orientability, connectedness, continuity, paper representations of topological surfaces, the nature of topological surfaces, and topological invariants?

19. Explain a cross-cap using the conceptual model described on pp. 79-82. Could you build such a model?

20. How can you interpret Martin Gardner's model in Figures 8-11 and described on pp. 83-84 as a projective plane? What is a projective plane? How does your imagination need to interpret the model to "see" the inherent self-intersection that is required in a projective plane?