Discussion Questions and Problems for "Map Coloring; Martin Gardner's Projective Plane & variations"

(Revision: April 23, 2018[1])

On 28 April 2018 *Math Counts* will discuss "Map Coloring; Martin Gardner's Projective Plane & variations"[2]. The following questions are based on Stephen Barr's fun book "Experiments in Topology". Your attempts to address them will help guide our discussion. If the book or these questions are unclear, please ask for help on the event page[2].

1. Inspired by the book, the questions that follow, or your own initiative, what additional experiments did you attempt? What did you learn from these supplemental experiments?

2. In Figure 1 on page 111, there is a map coloring of the torus that requires seven colors. Show that that map cannot be colored with six colors where a valid coloring requires that no two regions of the same color share a common segment of an edge.

3. In Figure 2 on page 111, there is a map coloring of the Möbius band that requires six colors. Show that that map cannot be colored with five colors.

4. In the PUZZLE on pages 118 and 119, color the map in Figure 17 subject to the constraint that you only have 24 square feet of RED, 24 square feet of YELLOW, and 16 square feet of GREEN.

5. On pages 82–85, Barr describes the Martin Gardner model of a projective plane (Figures 8–11). Do you understand the model? How does it work? How should your imagination interpret the model to "see" the inherent self-intersection that is required?

6. On page 83, Barr asks "why does a Möbius strip with only 1 twist give, when cut down the middle on its axis, a loop with 4 twists?" Does he mean 2 half-twists or 4 (does the explanation of a "twist" on page 82 clarify)? That seems to agree with the descriptions on pages 48 and 76, but the definition of a twist has changed? How can we count the number of twists in a topological surface? What is going on?

7. On page 85 in referring to the Gardner model in Figure 11, Barr asks "but the edge BA' is half in front and half behind the edge AB': is this allowable?" Can the required joints be made? If so, does it matter that the model is cut and distorted so each half goes separately to its join? Does that entail a discontinuity? Is there some disconnectedness? Is it OK?

8. In Figures 12 and 15 on pages 85–86, a Möbius band with cuts along half the centerline of both its length and width is experimented with. How are these cuts related to the Gardner model of the projective plane? What is the purpose of the experiments? What are the results of the experiments? Can you duplicate the results? Did you get both a two twist and a no twist cylinder? What is your interpretation of these results? What are the implications? Is it OK to cut slits in our models so long as we re-attach them with the correct connectivity as suggested by the definition of homeomorphism given in Chapter 1 of the book? What pitfalls must we be attentive to?

[1]Note: Please let me know of any difficulties. There may be a revised version correcting issues if any are found. Compiled by CJ Fearnley. `http://blog.CJFearnley.com`.

[2]`https://www.meetup.com/MathCounts/events/249730982/`

9. In Figure 19 on page 87 and in Figure 20 on page 88 and the associated text, two variations of the Gardner model are described. What is different about these models? Do they fix the problem with the cut flaps going to opposite sides? What are the results of the experiments? Do you also get a loop with two twists? Why is that the result? What does it mean?

10. In Figure 23 on page 90 and the associated text, a flat disk model of the projective plane is described. How can we interpret this as a projective plane? What are the implications of the dissection along aa'? Can every projective plane be so dissected? Why? Does that suggest that the projective plane is asymmetrical?

11. Do the results of these experiments suggest that the projective place is a symmetrical topological surface? Why? Why not?