Exploring Foldable Great Circle Geometries

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23 June 2009

Presentation to ISAMA 2009 (Eighth Interdisciplinary Conference of the International Society of the Arts, Mathematics, and Architecture) at the University at Albany, Albany, NY, USA, 22–25 June 2009

http://www.CJFearnley.com/isama.2009.presentation.pdf

Exploring Foldable Great Circle Geometries

Outline of the Talk

1. Introduction
   - What is Synergetics?
   - Synergetics Collaborative
2. Buckminster Fuller's 8 Foldable Great Circle Models
   - Introducing Fuller's Foldable Great Circle Models
   - Four Models Derived From the Vector Equilibrium
   - Three Models Derived From the Icosahedron
   - The Rhombic Triacontahedron
   - The Property of Foldability
3. Exploring Great Circle Foldability
   - Finding More Foldable Systems
   - Symmetrical Tessellations
   - Defining Foldability
   - A systematic study of foldability
   - Conclusion

CJ Fearnley: my background

- BA, Mathematics and Philosophy, Binghamton University
- Editor, R. Buckminster Fuller FAQ since May 1994
- Linux Systems Administrator, President, LinuxForce, Inc.
- Executive Director, Synergetics Collaborative
- Explorer in Universe

My foldable great circle geometry work started in Synergetics and consists of the mathematical development of questions that arose from my study of Synergetics.

Buckminster Fuller recommended starting all design and problem-solving work with the Universe, the whole, the Big Picture. So I'll start with Synergetics.

Notable Quotable

Dare to be Naïve!

— Buckminster Fuller

Comprehensive Thinking

Synergetics is the system of comprehensive thinking which R. Buckminster Fuller (1895–1983) introduced and began to formulate (primarily, in his two volume magnum opus, Synergetics: Explorations in the Geometry of Thinking, 1975, 1979).

Starting with the Universe: a major retrospective on Bucky's work now on display at the **Museum of Contemporary Art** in Chicago through July 5, 2009.

Synergetics Resources

- Both volumes of *Synergetics: Explorations in the Geometry of Thinking* are on-line at

  `http://www.rwgrayprojects.com/synergetics/synergetics.html`
- Amy Edmondson's book *A Fuller Explanation: The Synergetic Geometry of R. Buckminster Fuller*

  `http://www.angelfire.com/mt/marksomers/40.html`
- *Reading Synergetics: Some Tips*

  `http://www.cjfearnley.com/synergetics.essay.html`

Synergetics Collaborative (SNEC)

- The Synergetics Collaborative (SNEC) is a 501(c)(3) non-profit organization, founded in 2002, dedicated to bringing together a diverse group of people with an interest in Buckminster Fuller's Synergetics in face-to-face workshops, symposia, seminars, and other meetings to educate and support research and understanding of the many facets of Synergetics.
- Workshop: **A Synergetics Sampler**: July 11–12, 2009 in Oswego, NY

  `http://SynergeticsCollaborative.org/snec.announce.meeting.2009.07.html`
- Symposium: **Design Science: Nature's Problem Solving Method**: November 14–15, 2009 at RISD in Providence, RI

  `http://SynergeticsCollaborative.org/snec.announce.meeting.2009.11.html`
- Reports from past Workshops:

  `http://SynergeticsCollaborative.org/snec.meeting.2006.08.html`

  `http://SynergeticsCollaborative.org/snec.meeting.2005.07.html`
- YouTube Channel: `http://www.youtube.com/SynergeticsCollab`

Building Upon Bucky's Work

The **Synergetics Collaborative** is an educational and scientific non-profit working to build on Bucky's Synergetics by bringing together those who have been inspired by the work to develop it in new directions.

Folding the four great circles of the Cuboctahedron

- The four great circle model (formed by axes chosen from the centers of diametrically opposite triangular faces) can be easily folded into "bow-ties" without a compass

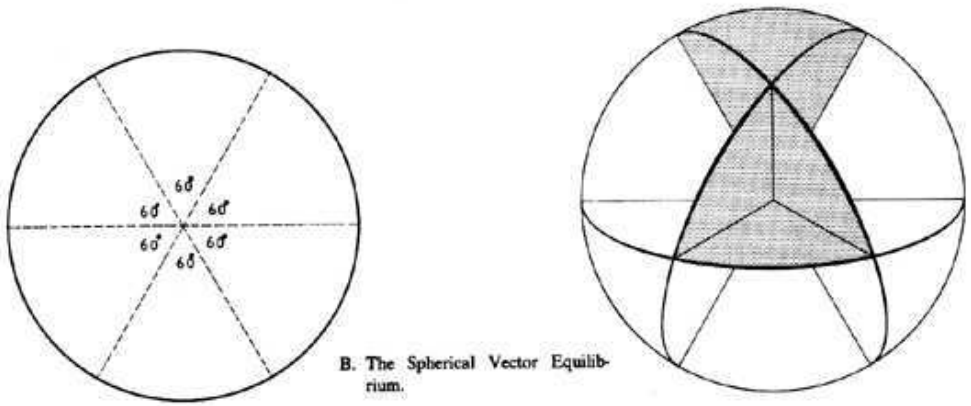

Fig. 455.11 Folding of Great Circles into Spherical Cube or Rhombic Dodecahedron and Vector Equilibrium: Bow-Tie Units:

A. This six-great-circle construction defines the positive-negative spherical tetrahedrons within the cube. This also reveals a spherical rhombic dodecahedron. The circles are folded into "bow-tie" units as shown. The shaded rectangles in the upper left indicates the typical plane represented by the six great circles.
B. The vector equilibrium is formed by four great circle folded into "bow-ties." The sum of the areas of the four great circles equals the surface area of the sphere. ($4\pi r^2$).

Bucky Fuller's Foldable Great Circle Models

- In Synergetics, Fuller documented 8 great circle tessellations of the sphere which can be constructed by folding whole discs (such as paper plates) into conical modules which dovetail into a model of the whole sphere.
- Four (4) of them are characterized by the equators of spin traced by sets of rotating axes each of whose endpoints are diametrically opposite pairs of topological elements (vertices, faces, & edges) of the VE or Vector Equilibrium (what is traditionally called the "Cuboctahedron").

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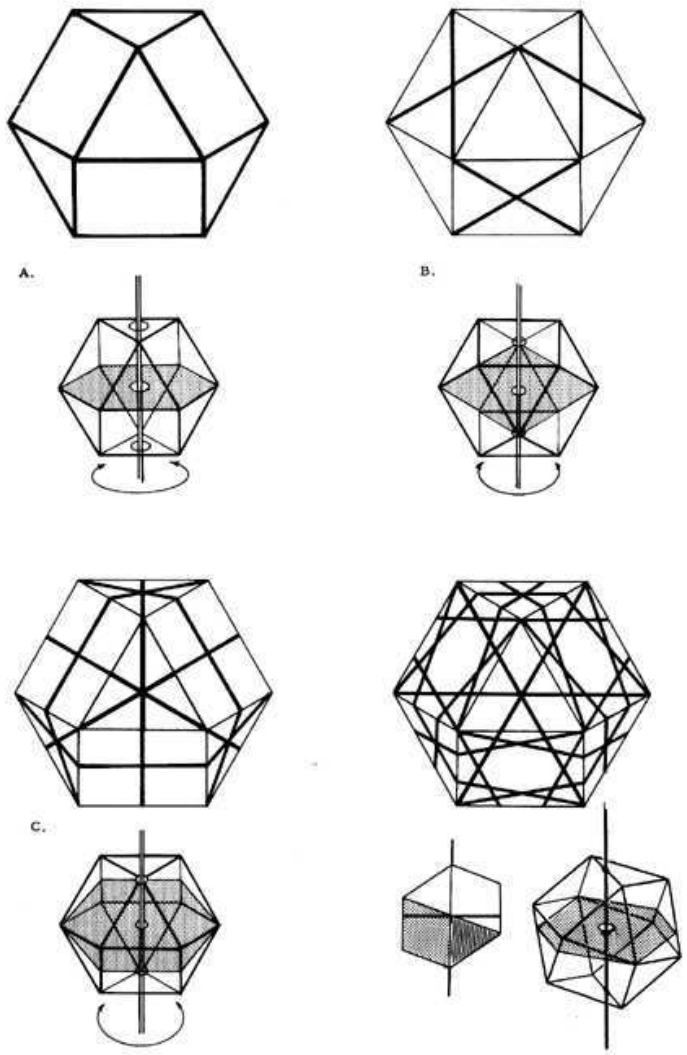


Fig. 450.11A Axes of Rotation of Vector Equilibrium:

A. Rotation of vector equilibrium on axes through centers of opposite trianglar faces defines four equatorial great-circle planes.
B. Rotation of the vector equilibrium on axes through centers of opposite square faces defines three equatorial great-circle planes.
C. Rotation of vector equilibrium on axes through opposite vertexes defines six equatorial great-circle planes.
D. Rotation of the vector equilibrium on axes through centers of opposite edges defines twelve equatorial great-circle planes.

Folding the three great circles of the VE

- The three great circle model (formed by axes chosen from the centers of diametrically opposite square faces) has each edge doubled

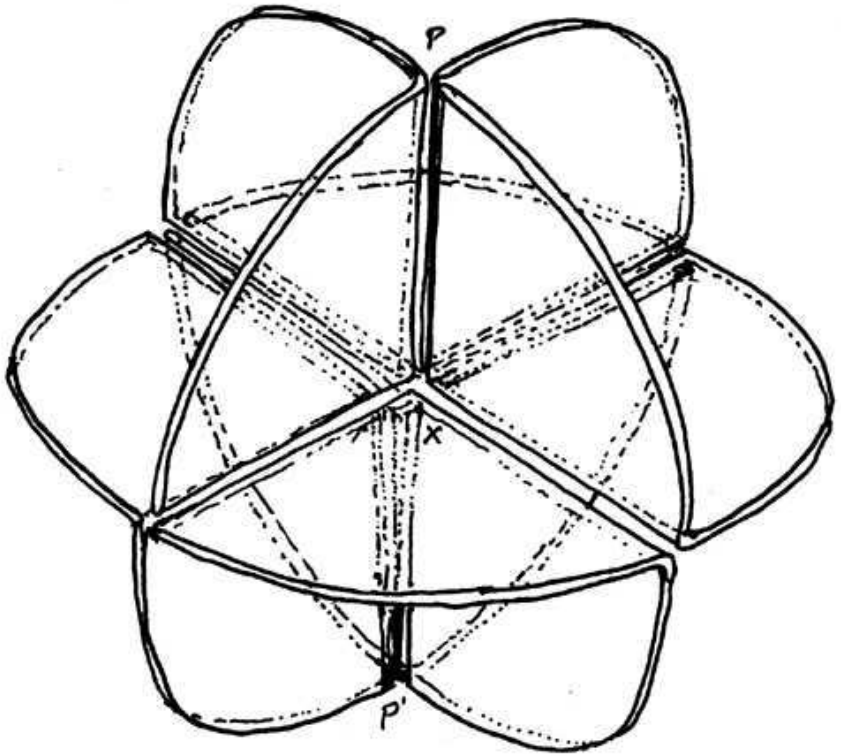
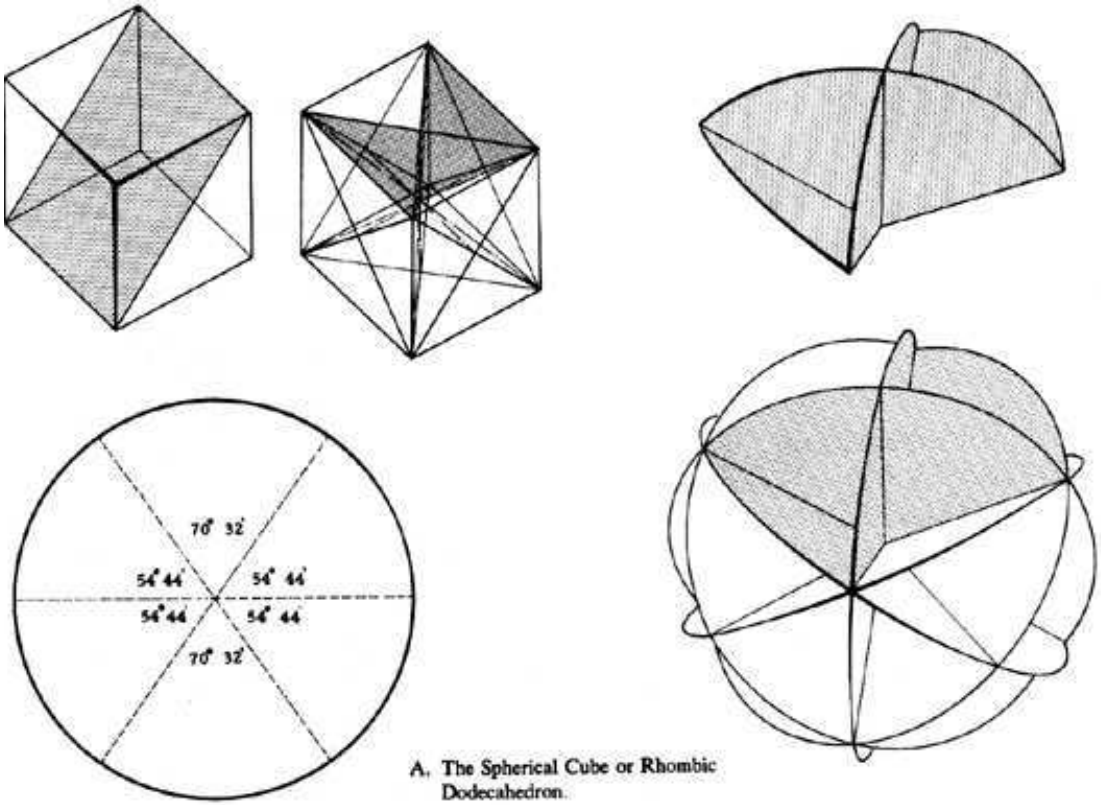


Fig. 835.10 Six Great Circles Folded to Form Octahedron.

Folding the six great circles of the VE

This six-great-circle construction defines the positive-negative spherical tetrahedrons within the cube. This also reveals a spherical rhombic dodecahedron. The circles are folded into "bow-tie" units as shown. The shaded rectangles in the upper left indicates the typical plane represented by the six great circles.



A. The Spherical Cube or Rhombic Dodecahedron.

Folding the twelve great circles of the VE

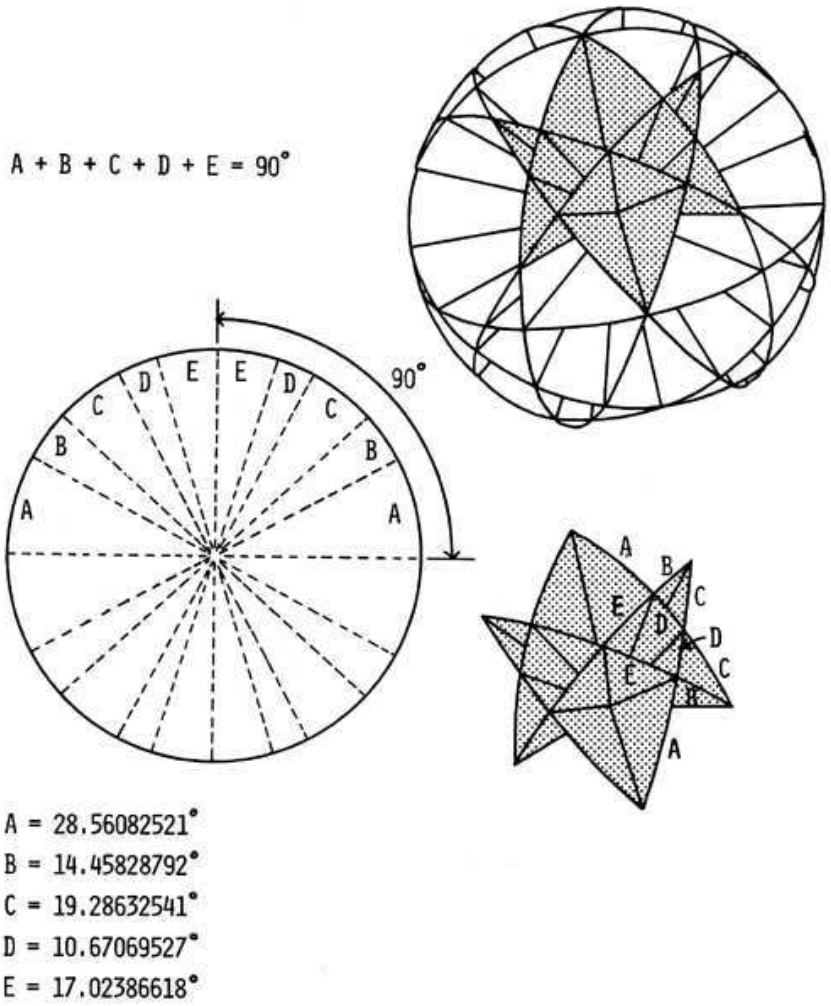


Fig. 450.10 The 12 Great Circles of the Vector Equilibrium Constructed from 12 Folded Units (Shwon as Shaded).

Schwarz Triangle

A **Schwarz triangle** is a spherical triangle that can tile (tessellate) a sphere by reflection across its edges.

Note: Fuller's Lowest Common Denominator (LCD) triangles are the faces cut by the planes of symmetry.

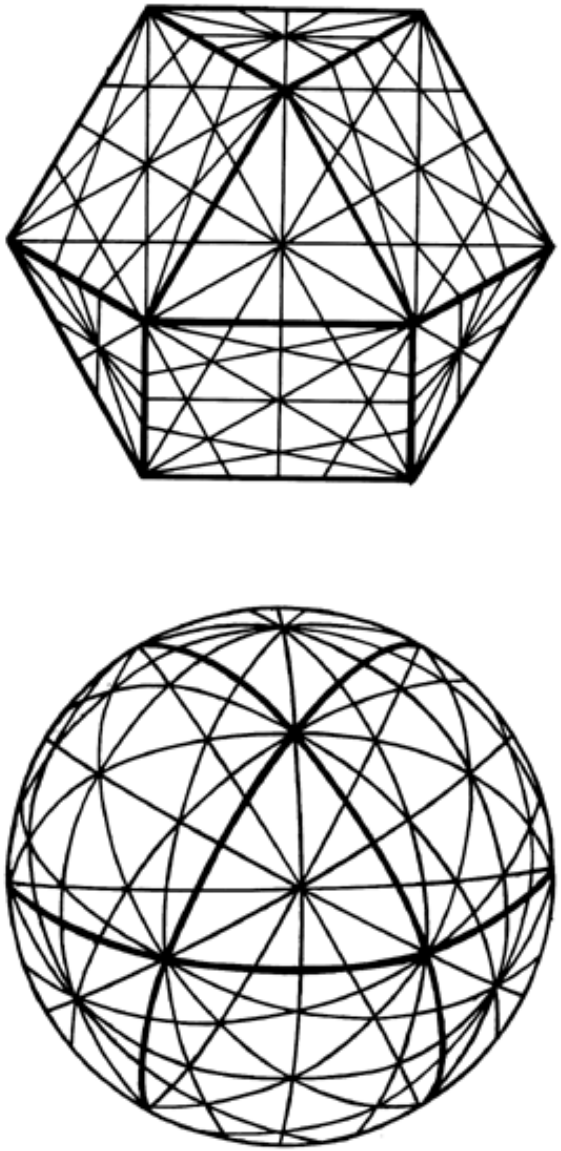


Fig. 450.11B Projection of 25 Great-Circle Planes in Vector Equilibrium Systems: The complete vector equilibrium system of 25 great-circle planes, shown as both a plane faced-figure and as the complete sphere (3 + 4 + 6 + 12 = 25). The heavy lines show the edges of the original 14-faced vector equilibrium.

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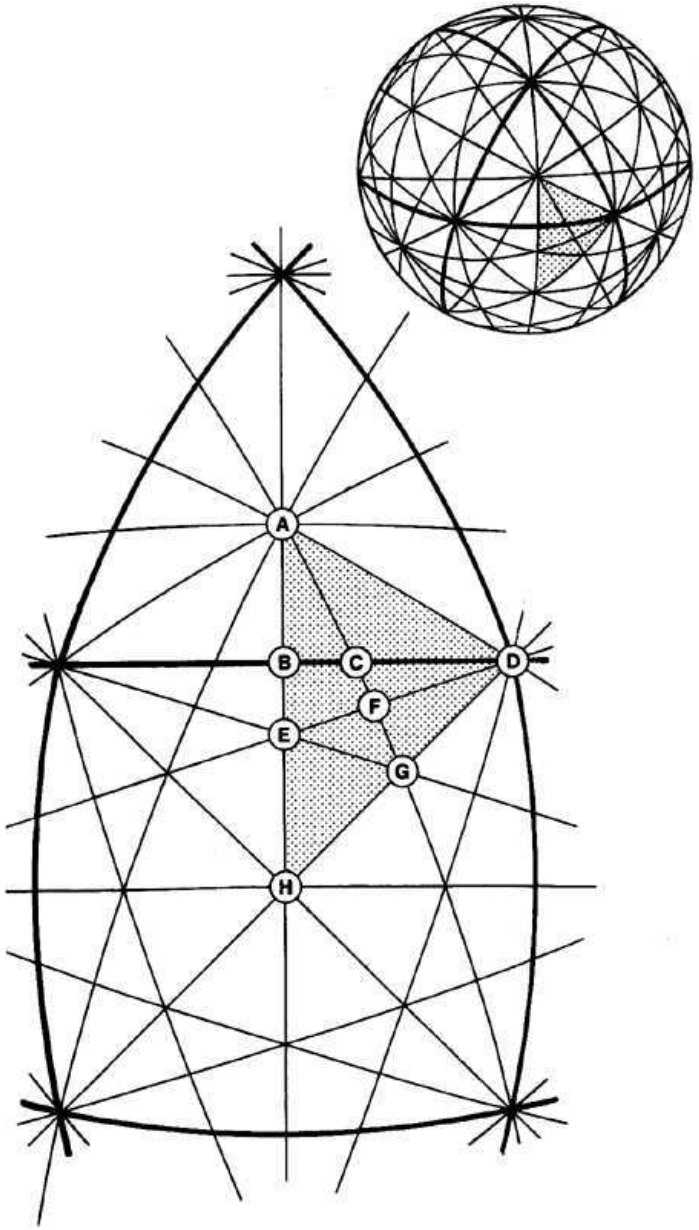


CENTRAL ANGLES		
19.47122063°	AB	19° 28' 16.394"
35.26438968	AD	35 15 51.803
22.20765430	AC	22 12 27.555
10.89339465	BC	10 53 36.221
19.10660535	CD	19 06 23.779
10.02498786	BE	10 01 29.956
6.35317091	CF	6 21 11.415
14.45828792	EF	14 27 29.837
17.02386618	FD	17 01 25.918
19.28632541	EG	19 17 10.771
10.67069527	FG	10 40 14.503
25.23940182	EH	25 14 21.847
26.56505118	HG	26 33 54.184
18.43494882	GD	18 26 5.816
31.48215410	DE	31 28 55.755
30.	BD	30 00 00
45.	DH	45 00 00
54.73561031	AH	54 44 8.197

FACE ANGLES		
30.°	BAC	30° 00' 00.000"
30.	CAD	30 00 00.000
90.	ABC	90 00 00.000
61.87449430°	ACB	61 52 28.179
118.1255057	ACD	118 7 31.821
35.26438968	ADC	35 15 51.803
90.	EBC	90 00 00.000
118.1255057	BCF	118 7 31.821
73.22134512	BEF	73 13 16.842
80.40593179	CFE	80 24 21.354
61.87449430	FCD	61 52 28.179
19.47122063	CDF	19 28 16.394
99.59406821	CFD	99 35 38.646
73.22134512	HEG	73 13 16.842
65.90515745	EGH	65 54 18.567
45.	EHG	45 00 00.000
99.59406821	EFG	99 35 38.646
33.55730977	FEG	33 33 26.315
48.18968511	FGE	48 11 22.866
80.40593179	GFD	80 24 21.354
35.26438969	FDG	35 15 51.803
65.90515745	FGD	65 54 18.567

Fig. 453.01 Great Circles of Vector Equilibrium Define Lowest Common Multiple Triangle: 1/48th of a Sphere: The shaded triangle is 1/48th of the entire sphere and is the lowest common denominator (in 24 rights and 24 lefts) of the total spherical surface. The 48 LCD triangles defined by the 25 great circles of the vector equilibrium are grouped together in whole increments to define exactly the spherical surface areas, edges, and vertexes of the spherical tetrahedron, spherical cube, spherical octahedron, and spherical rhombic dodecahedron. The heavy lines are the edges of the four great circles of the vector equilibrium. Included here is the spherical trigonometry data for this lowest-common-denominator triangle of 25-great-circle hierarchy of the vector equilibrium.

Folding the great circles of the Icosahedron

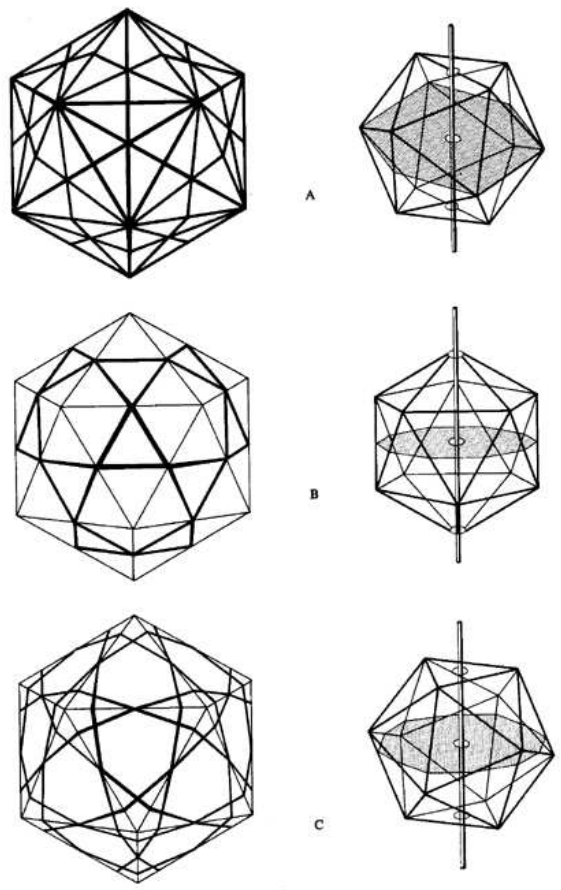


Fig. 457.30A Axes of Rotation of Icosahedron:

A. The rotation of the icosahedron on axes through midpoints of opposite edges define 15 great-circle planes.
B. The rotation of the icosahedron on axes through opposite vertexes define six equatorial great-circle planes, none of which pass through any vertexes.
C. The rotation of the icosahedron on axes through the centers of opposite faces define ten equatorial great-circle planes, which do not pass through any vertexes.

Folding the 6 & 15 great circles of the Icosahedron

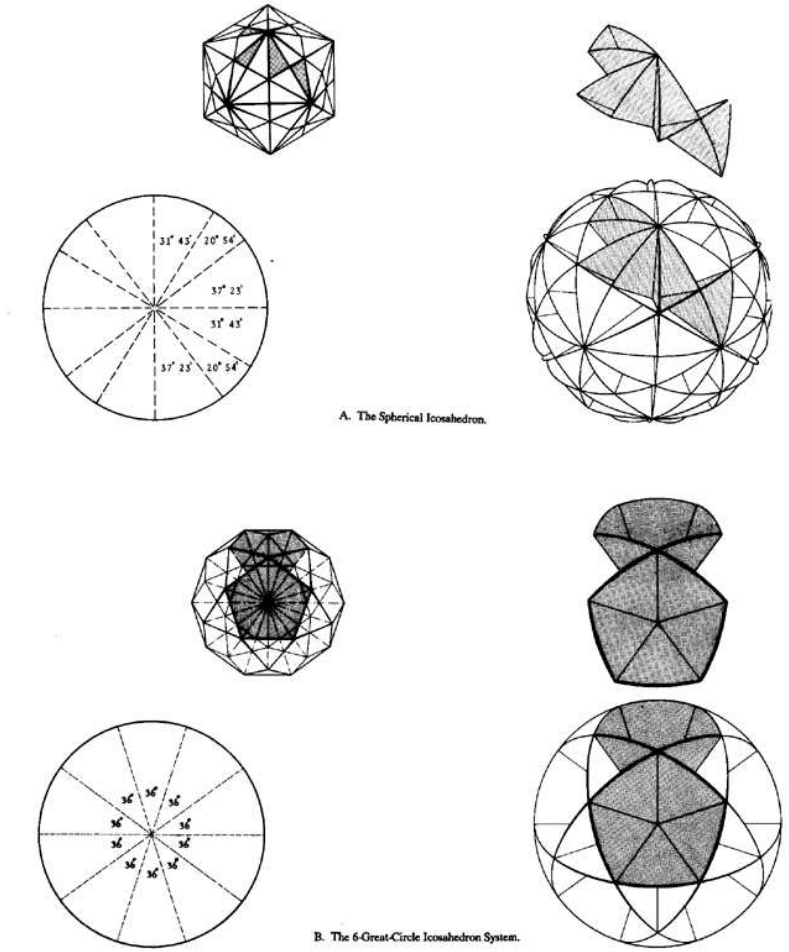


Fig. 458.12 Folding of Great Circles into the Icosahedron System:

A. The 15 great circles of the icosahedron folded into "multi-bow-ties" consisting of four tetrahedrons each. Four times 15 equals 60, which is 1/2 the number of triangles on the sphere. Sixty additional triangles inadvertently appear, revealing the 120 identical (although right- and left- handed) spherical triangles, which are the maximum number of like units that may be used to subdivide the sphere.
B. The six great-circle icosahedron system created from six pentagonal "bow-ties."

Folding the 10 great circles of the Icosahedron

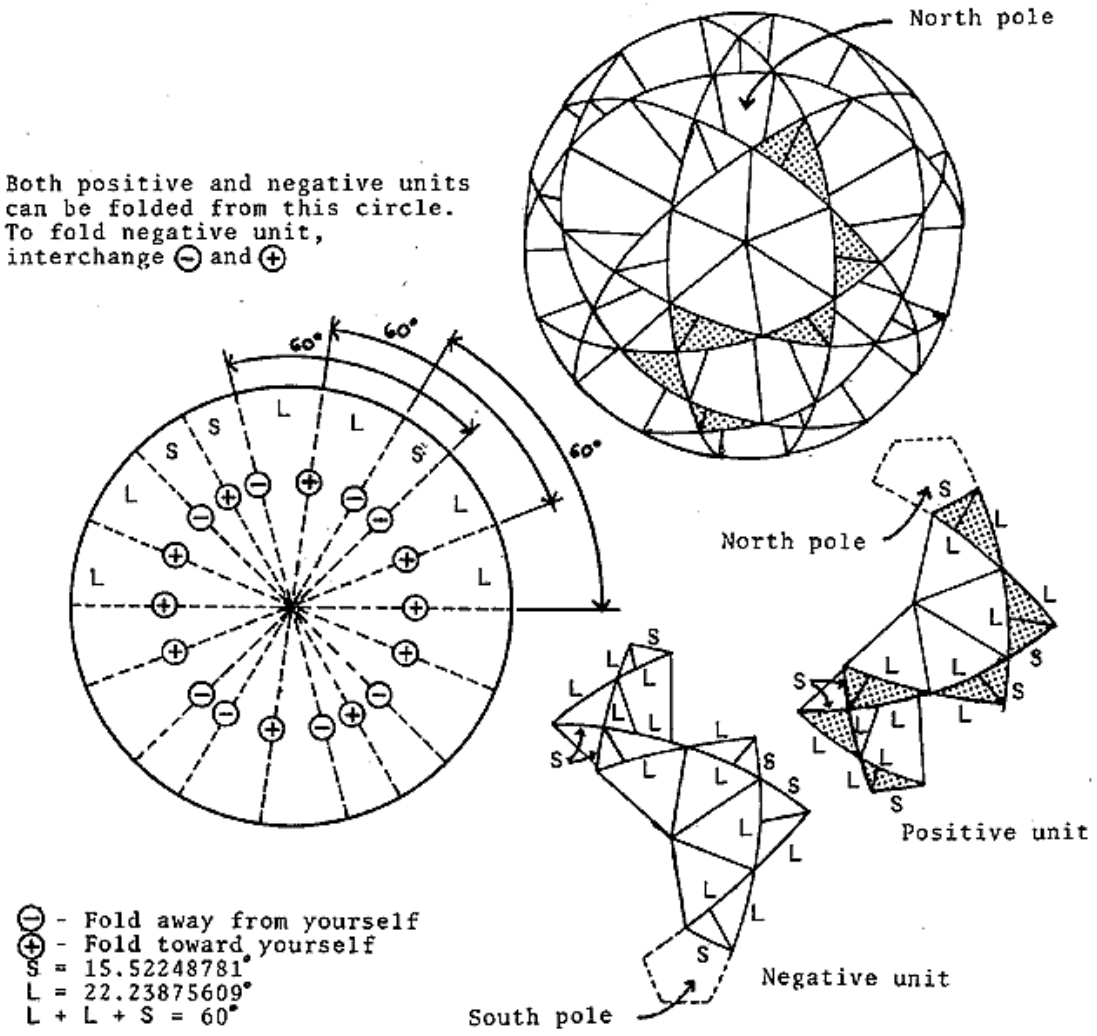


Fig. 455.20 The 10 great circles of the Icosahedron Constructed from 10 folded units (5 positive units + 5 negative units).

The 31 great circles of the Icosahedron

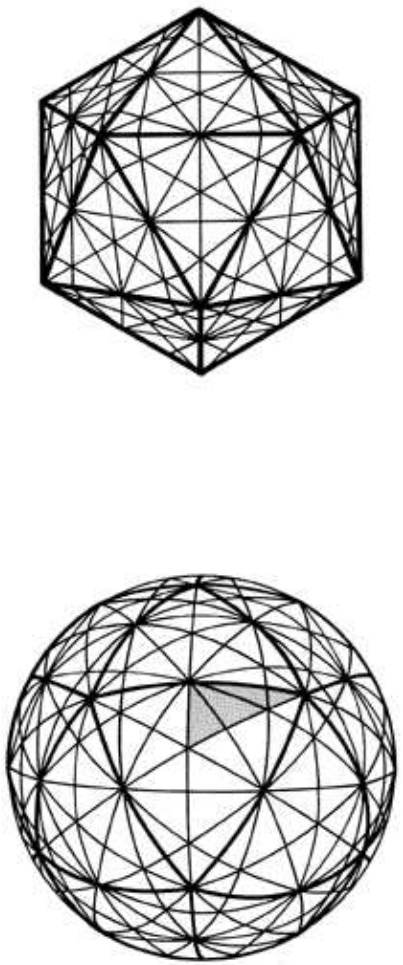
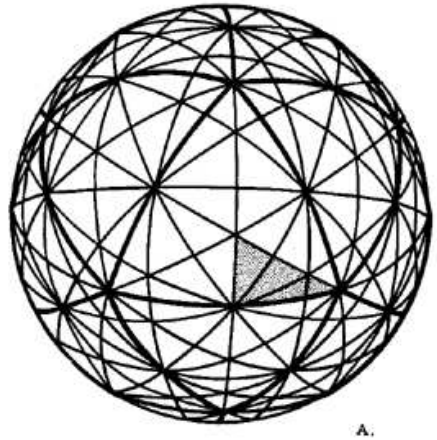


Fig. 457.30B Projection of 31 Great-Circle Planes in Icosahedron System: The complete icosahedron system of 31 great-circle planes shown with the planar icosahedron as well as true circles on a sphere (6+10+15=31). The heavy lines show the edges of the original 20-faced icosahedron.



A.

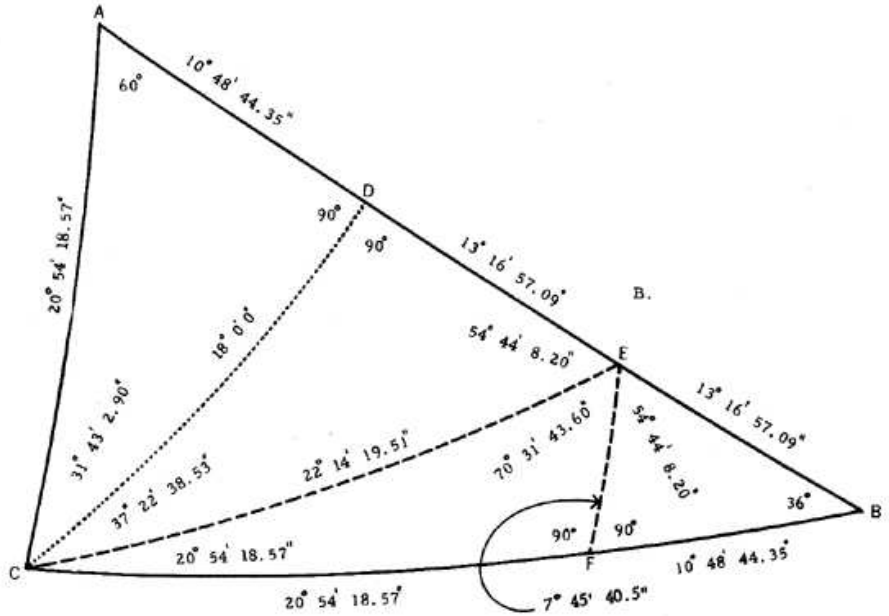


Fig. 901.03 Basic Right Triangle of Geodesic Sphere: Shown here is the basic data for the 31 great circles of the spherical icosahedron, which is the basis for all geodesic dome calculations. The basic right triangle as the lowest common denominator of a sphere's surface includes all the data for the entire sphere. It is precisely 1/120th of the sphere's surface and is shown as shaded on the 31-great-circle- sphere (A). An enlarged view of the same triangle is shown (B) with all of the basic data denoted. There are three different external edges and three different internal edges for a total of six different edges. There are six different internal angles other than 60° or 90°. Note that all data given is spherical data, i.e. edges are given as central angles and face angles are for spherical triangles. No chord factors are shown. Those not already indicated elsewhere are given by the equation 2 sin(theta/2), where theta is the central angle. Solid lines denote the set of 15 great circles. Dashed lines denote the set of 10 great circles. Dotted lines denote the set of 6 great circles.

Bucky's 15 great circles model (with doubled edges)

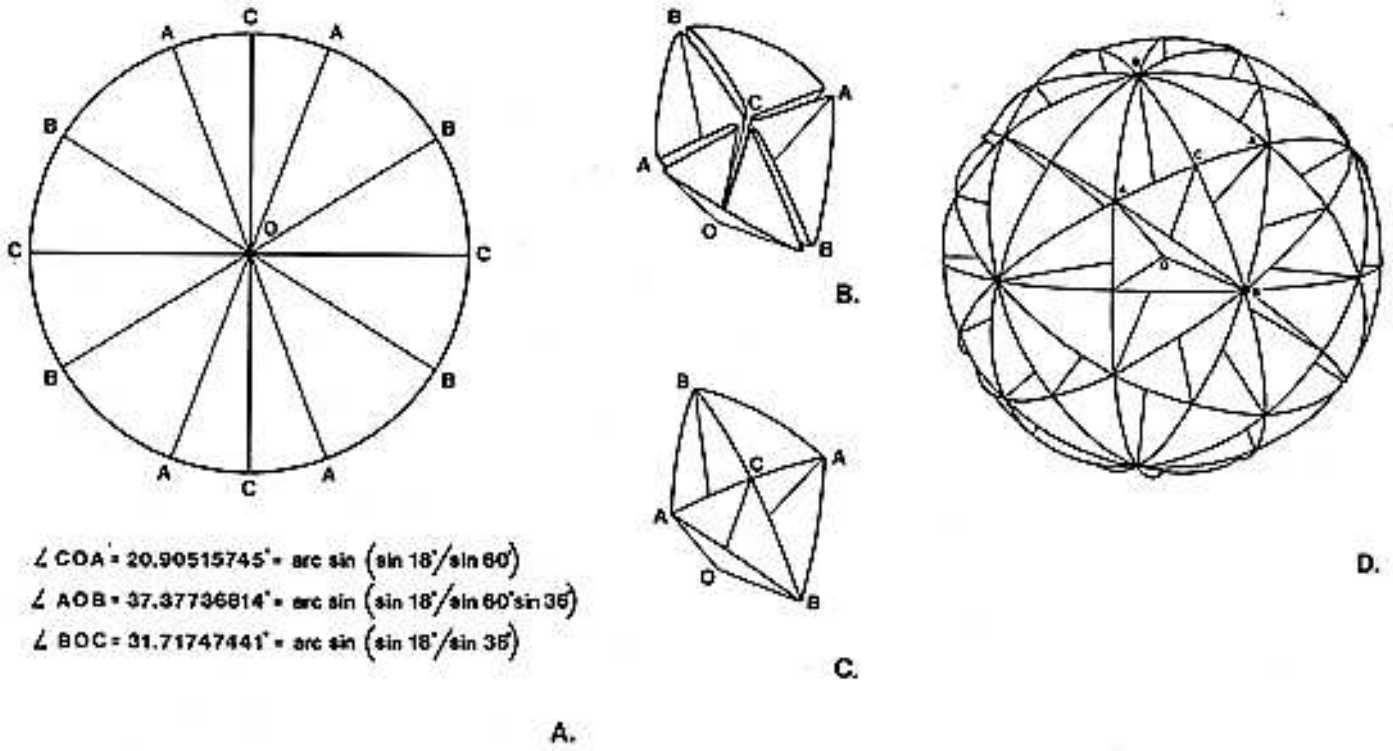


Fig. 986.502 Thirty Great-circle Discs Foldable into Rhombic Triacontahedron System: Each of the four degree quadrants, when folded as indicated at A and B, form separate T Quanta Module tetrahedra. Orientations are indicated by letter on the great-circle assembly at D.

Bucky's modules dovetail to form great circle nets

The remarkable thing about these models is how the modules (folded circles) dovetail together to form complete great circle tessellations.

- One might think that an arbitrary folding of a circle would not typically dovetail with other arbitrary folding to compose a great circle tessellation of the sphere.
- One would expect an arbitrary decomposition of a great circle net into disjoint Eulerian circuits (closed subgraphs that visit each edge only once) to rarely have exactly 360° of arc.
  - An *Eulerian graph* is one where a closed walk "visits" each edge exactly once.
  - A connected graph is *Eulerian* if and only if every vertex has even degree, that is, an even number of edges meet at each vertex (Euler 1736).
- So these models are mathematically special in some way

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Historical Context

- Fuller developed these foldable great circle models as part of his Synergetics research which led to his most famous invention, the geodesic dome. They were published in Synergetics in 1975 and 1979.
- I built them in college in 1988 and 1989.
- Jeannie Moberly and I investigated the 25 great circle aggregate model and wrote a paper and a presentation for Bridges 2004
  - http://www.cjfearnley.com/supercircles.slides.01.pdf
- I last presented this work at the American Mathematical Society's 2008 Spring Eastern Meeting (#1036)
  - http://www.cjfearnley.com/folding.great.circles.2008.pdf

Summary of Fuller's Foldable Models

- Fuller documented 8 Models that form great circle tessellations of the sphere using modules built by folding whole discs.
- Fuller's modules effectively partition a great circle graph on the sphere into disjoint Eulerian circuits of exactly 360°.
- Since Fuller gave us eight (8) models, could foldability be commonplace?
- Or is the foldability property rare?
- Which great circle tessellations are foldable and why?

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Identifying Additional Foldable Tessellations

- The simplest foldable great circle net consists of a single unfolded circle
- n discs folded in half and mated along their diameters generates an "infinite" class of foldable models (the axes of spin for these great circles all lie in one plane which is, of course, another great circle).
  - Of course, the arcs must be fixed at the "correct" angles to ensure we have great circles and not just a set of half-circles.
  - So if these foldables are made of paper plates and bobby pins, they will be "unstable" and hence cannot be physically realized (unless some way to fix the angle is developed either physically or in a computer simulation or using abstraction).

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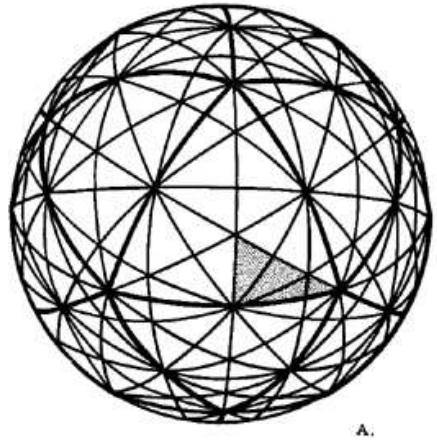
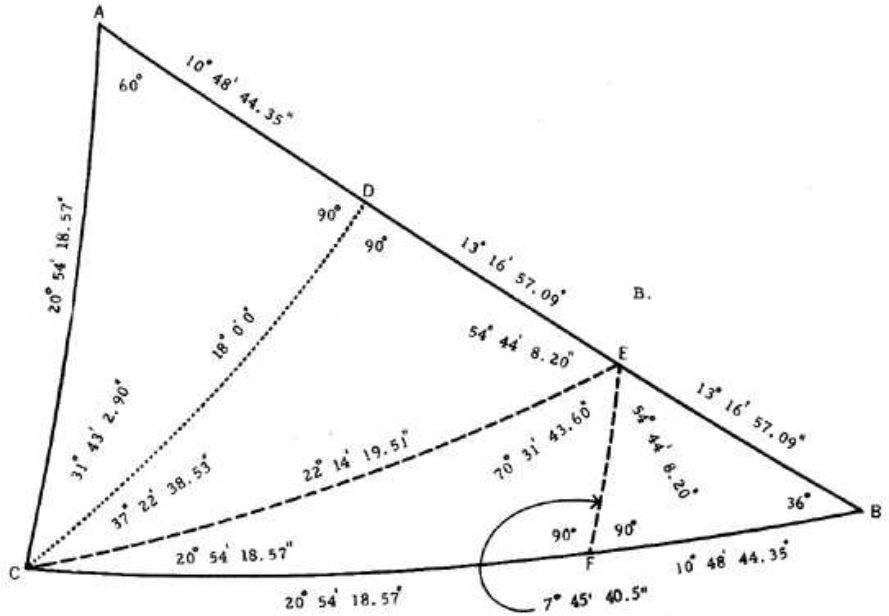
Are the secondary great circles of the icosa Foldable?

Is the set of equators formed by axes defined by diametrically opposite vertices D, E, and F (each of which is defined by points of intersections on the boundary of the Schwarz triangle in the 31 great circles aggregate of the icosa) foldable?



A.



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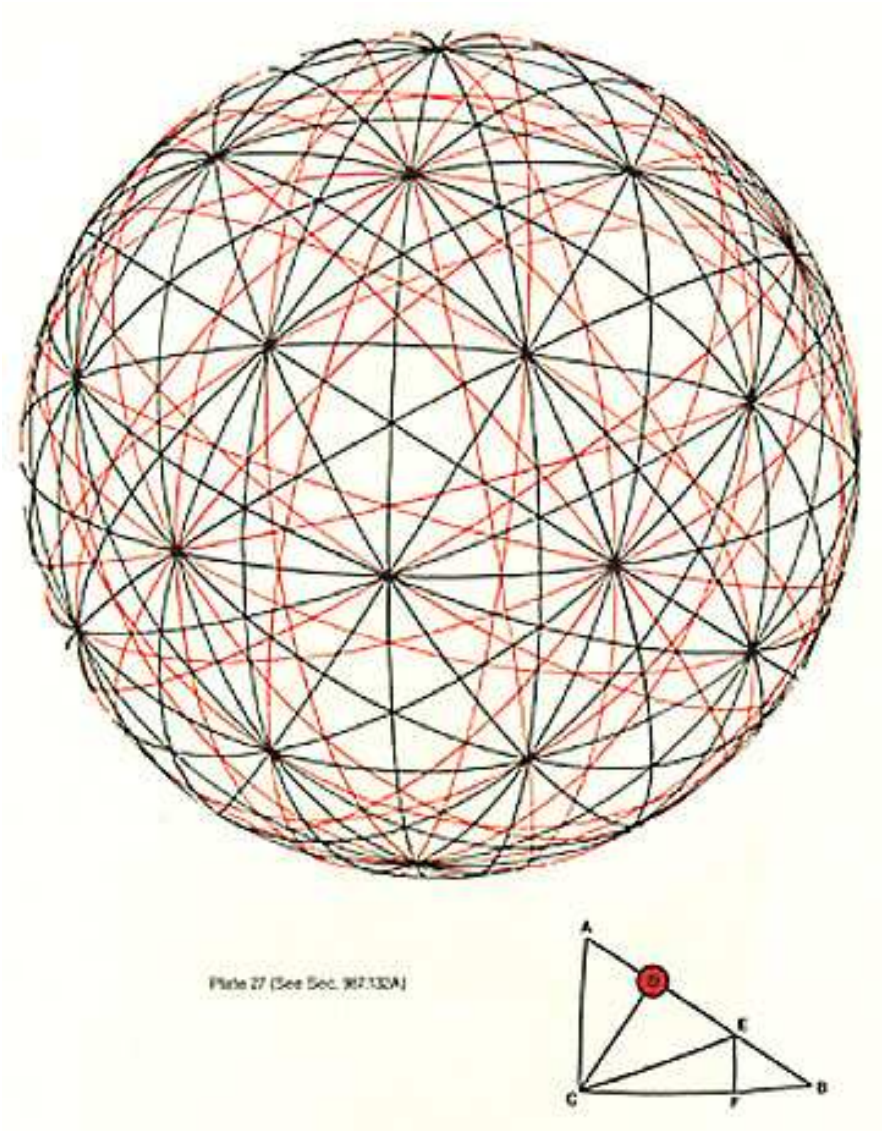
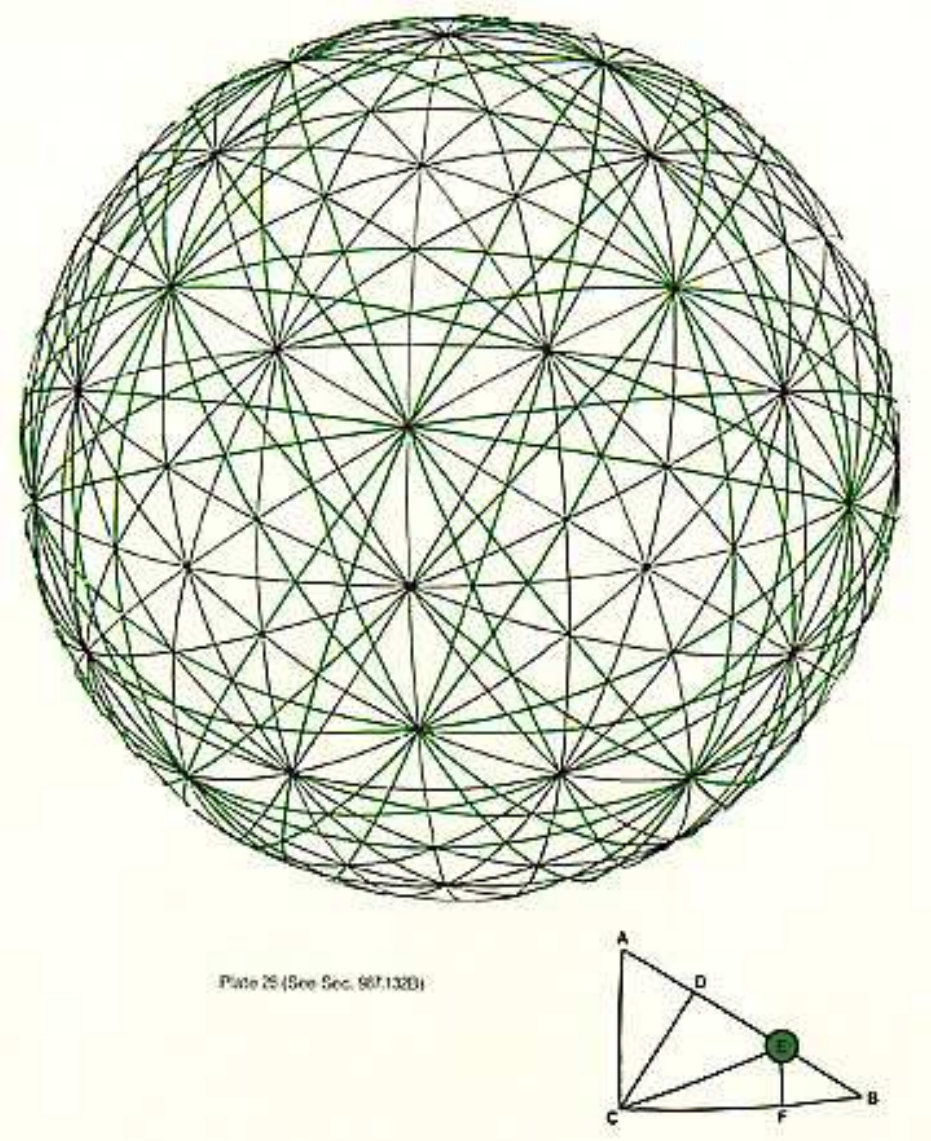
The secondary GCs of the icosa are Foldable



The secondary GCs of the icosa are Foldable



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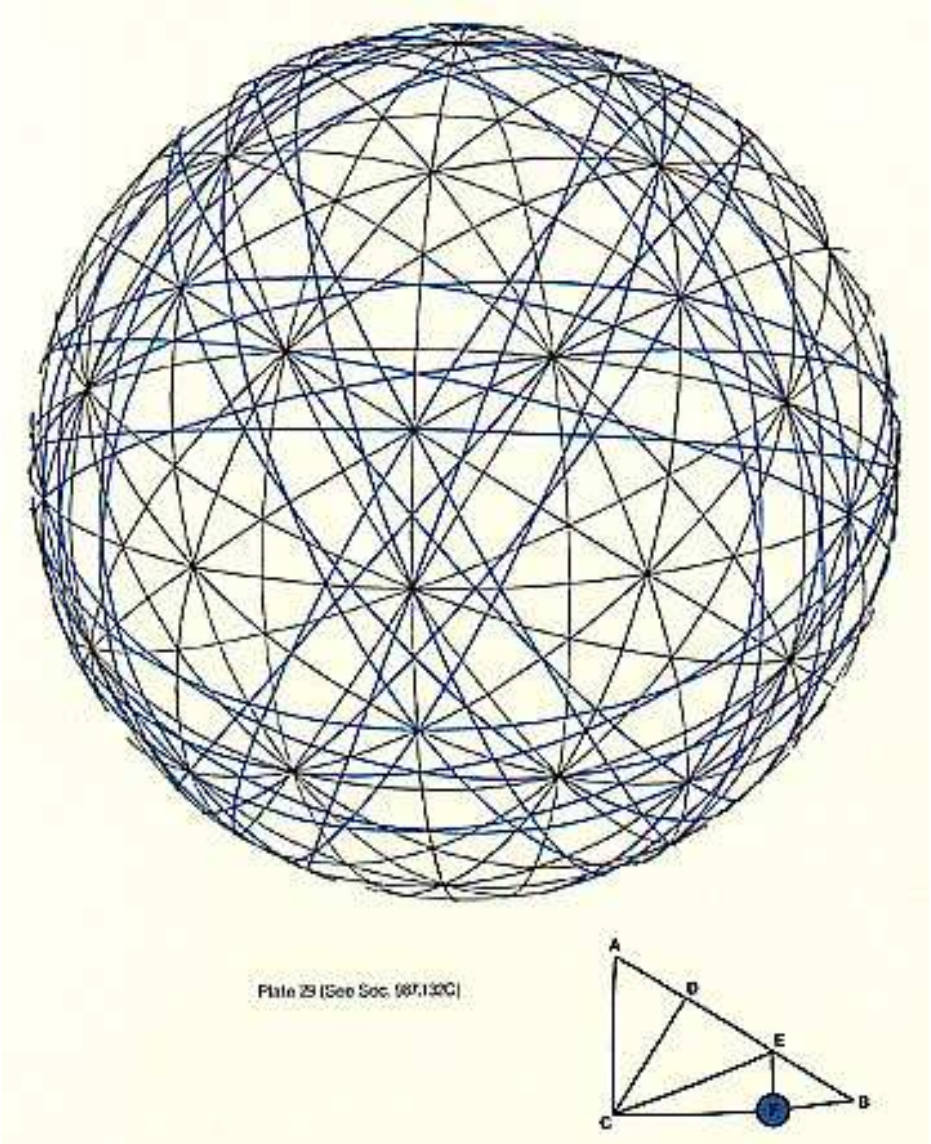
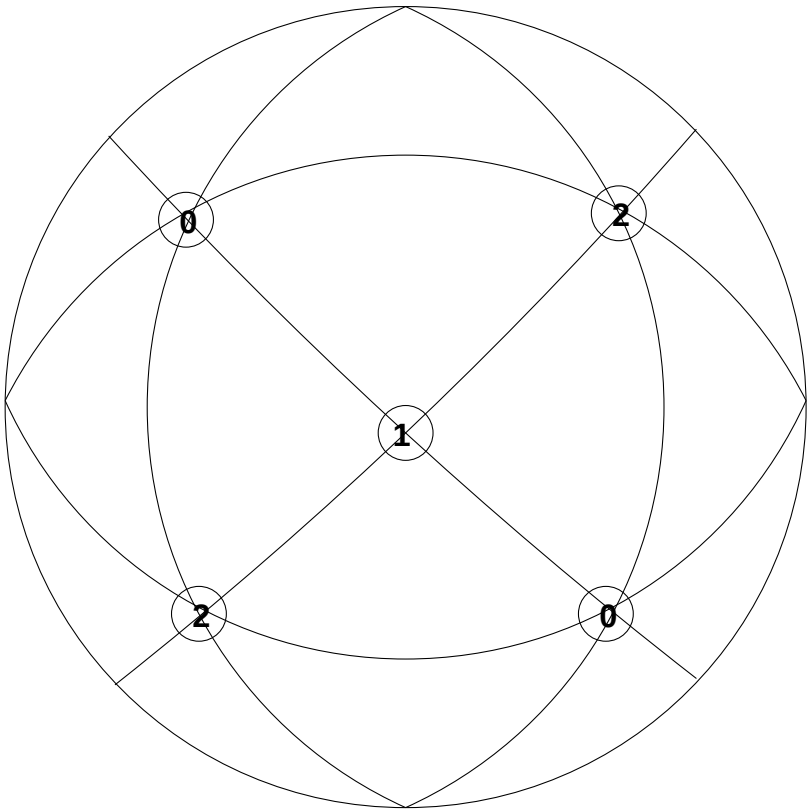


Might the foldability property be related to symmetry?

Symmetry: I then asked if simple symmetries might generate more foldable tessellations? So I looked in Coxeter's *Regular Polytopes* at the *"lines of symmetry"* for the regular hedra. These are simply the spherical great circle nets cut by the planes of symmetry for each regular hedron. Are these geometries foldable?

Some "Lines of Symmetry" Models are NOT Foldable

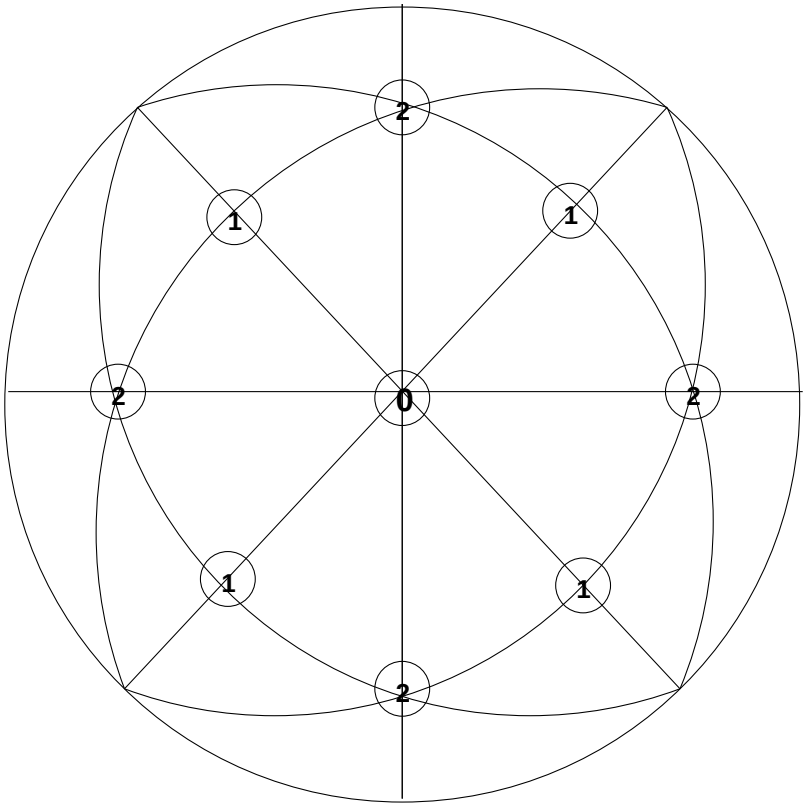
Foldable



6 Great Circles (lines of symmetry)

Spherical Tetrahedron

Not Foldable



9 Great Circles (lines of symmetry)

Octahedron / Hexahedron

The 9 "Lines of Symmetry" of the Octa are NOT Foldable

Let's examine the 9 lines of symmetry great circle net for the hexahedron / octahedron net. There are 48 (**0–1–2**) Schwarz triangles in the system. Only 24 of them are needed for our modules (a figure/ground relationship). Evidently any Eulerian path will consist of *n* such modules. There must be $\frac{9}{24}$ (or 135°) of a disc in each Schwarz triangle which is incommensurable with 360°. Therefore, the geometry is NOT foldable.

Using spherical trig, we can calculate the arcs: **0–1** is 45°, **0–2** is $54^\circ\, 44'\, 8.2''$, and **1–2** is $35^\circ\, 15'\, 51.8''$. Again, we see that 135° is incommensurable with 360°.

A Proposed Classification of Great Circle Foldability

Sub Foldability modules consist of arcs (less than 360°) cut from whole discs. Any model with great circle arcs can be built in this way.

Integral Foldability modules consist of whole circles with exactly 360° of arc. The number of modules n exactly equals the number of great circles N (that is, $n = N$).

Rational Foldability Each fold generates a vertex in the spherical net and $n = NF$ (i.e., $N \mid n$) where F is an integer. That is, some arcs are duplicated.

- Another possibility is to allow different types of modules (n_i of each type) each of which contributes a different fraction F_i of a whole disc. So that $\sum n_i F_i = N$. But I haven't found any yet.

A Proposed Classification of Great Circle Foldability (continued)

Irrational Foldability Any spherical tessellation can be folded using the techniques in *Geometric Folding Algorithms: Linkages, Origami, Polyhedra by Demaine and O'Rourke (2007).*

Super Foldability *Splicing in arcs to build modules with more than* 360° *is possible. Again this allows folding arbitrary spherical tessellations.*

My Main Question

Which great circle tessellations are (integral and rational) foldable?

My definition tells how to decide if a great circle net is foldable by calculating the amount of arc in each Eulerian circuit partitioning of the net (using either simple divisibility tests or spherical trig).

Is it possible to specify a criterion that will distinguish the foldables from the unfoldables based on some intrinsic property of the geometry (for example like certain types of symmetry or particular relationships of the axes of spin or some other quality of spherical geometry)?

In short, my quest is to find a theory of great circle foldability.

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Conjecture

Conjecture If the number of great circles equals the number of fundamental regions in a symmetrical tessellation of the sphere, then it is integrably foldable.

Note: this is clearly a necessary condition, but is it sufficient?

A systematic study of foldability

Following a lead from Robert Gray, we note that one can choose one of the great circles as an equator and "hide" the mirror image on the other side "behind the page". Therefore, one can model a great circle net as a simple circle with lines cutting through it.

Alternatively, using a Schlegel diagram (by projecting the spherical net through a point beyond one of its openings, or faces) might provide a means to track the arc lengths on a sheet of paper to search for nets that are foldable.

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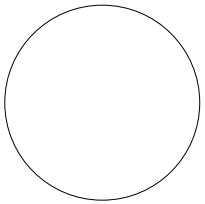
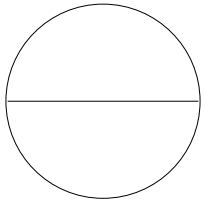
Diagramming Great Circle Nets

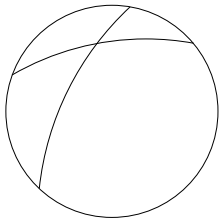
Study of Foldability in Simple Spherical Great Circle Nets



One Great Circle

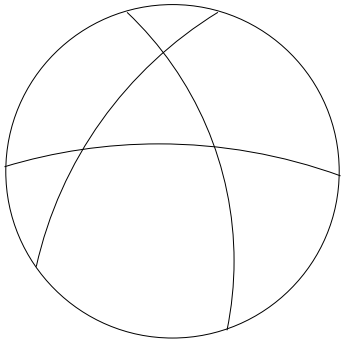


Two Great Circles

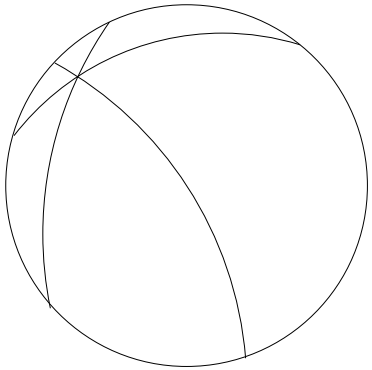


Three Great Circles

Parts of interior arcs sum to 180 degrees
"Slide" the arcs around to explore each possible 3 great circle net.



Four Great Circles
Are all variations foldable?



Four Great Circles

With fewer intersections
The geometry changes
Never foldable???

Open Questions

- How to define a theory for all integral (and rational) foldable spherical tessellations?
- Are the secondary great circles of the VE also foldable?
- Are all three great circle nets foldable? [I suspect yes.]
- Are all four great circle nets foldable? [I suspect no.]
- Does there exist a spherical great circle net with the property that several different modules dovetail to produce the whole model? [I suspect the 15 great circles of the icosa might have this property.]

Applications

- This whole project is a demonstration of the interesting, subtle, and unexplored geometry that lies just behind the scenes in Buckminster Fuller's Synergetics.
- The foldable great circle geometries provide insight into how whole systems can be omni-directionally dissected into modules which could provide insights into the structure of radiation and central projections, and modular building systems. For example, the dissections may be relevant to organizing structures in string theory or architecture.
- Educationally the foldable great circle models provide motivation for spherical trigonometry, graph theory, and an understanding of spherical geometry, in general.
- A model of Organic development from 2-D to 3-D by folding.
- Fascinating objects d'art to hang in your living room!

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Conclusion

Buckminster Fuller's magnum opus, Synergetics, is pregnant with unexplored mathematical themes which, to date, the mathematics community has largely ignored. My work on foldable great circle geometries provides an example of how the type of basic mathematical questions implicit in Synergetics can result in studies of interest to artists, architects, mathematicians and students.

The quest to understand foldable great circle geometries will continue as a theory for great circle foldability has remained elusive!

Thank You

Thank You!

Any Questions?

http://www.CJFearnley.com/isama.2009.presentation.pdf